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On Fibrewise star topological spaces

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عن الفضاءات التبولوجية النجمية الليفية

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Abstract:
This paper introduces fibrewise star topological spaces, a new concept in fibrewise topology. We explore their fundamental properties and connections to modern applications, such as adaptive network systems. By studying projections and neighborhood structures in these spaces, we establish key theoretical results while demonstrating their potential for modeling multi-domain problems. The work bridges abstract topology with practical computational frameworks, offering a foundation for future research in compactness, connectedness, and real-world implementations.

Keywords: Fibrewise topological space, Star topology, Fibrewise star topological space.

الملخص
تقدم هذه الدراسة مفهوم الفضاءات التبولوجية النجمية النسبية، وهو مفهوم جديد ضمن إطار التبولوجيا النسبية. وتتناول الورقة الخصائص الأساسية لهذه الفضاءات، بالإضافة إلى استكشاف ارتباطها بالتطبيقات المعاصرة، كانظمة الشبكات التكيفية. ومن خلال تحليل الإسقاطات والبنى المجاورة ضمن هذه الفضاءات، يتم التوصل إلى نتائج نظرية جوهرية، مع إبراز قابليتها لنمذجة المشكلات متعددة المجالات. كما يربط هذا العمل بين التبولوجيا المجردة والأطر الحاسوبية التطبيقية، مما يوفر أساسا علميا لبحوث مستقبلية في مفاهيم الانضغاط، والاتصال، والتطبيقات الواقعية.

الكلمات المفتاحية: الفضاء التبولوجي الليفي، التبولوجي النجمي، الفضاء التبولوجي النجمي الليفي.

Introduction and Preliminaries:

Inspired by the idea that the objective of General Topology is the study of continuous functions, a branch known as *continuous function topology* or *fibrewise topology* [1] has emerged. This research focuses on the category of fibrewise sets over a given base set. If the base set is denoted by B, then a fibrewise set over B consists of a set X along with a projection function $P:X\rightarrow B$. For each point $b\in B$, the *fibre* over b is the subset $Xb=P-1(b)$ of X. Note that fibres may be empty since P is not required to be surjective. Additionally, for any subset $B'\subseteq B$, we consider $XB'=P-1(B')$ as a fibrewise set over B' with the projection induced by P [2]. In large-scale platforms such as e-commerce and online marketing, there is a growing need for a unified recommendation model capable of efficiently handling multiple domains. Traditional approaches either rely on a single model for all domains or employ separate models for each domain - both of which have significant limitations.

The star topology Adaptive Recommender (STAR) model, introduced in a paper 1at (IKM2021), [3], presents a new solution by combining common and domain-specific components with a star topology architecture so that the model can learn commonalities as well as unique features across different domains. Since, The core idea of STAR is its star network architecture, where each domains model consists of two parts: i-Shared Centered Network: learns universal user behaviour patterns that occur in all domains. Domain-Specific Network: Relatively learns specificbehaviors and attributes of each individual domain.[4] These are both blended by element-wise multiplication of their respective weights within every one of the neural networks, layers. This allows the model to dynamically adapt itself to each individual domain while still benefiting from cross-domain knowledge sharing. multiplication of their respective weights (Sheng et al., 2021) within every one of the neural networks' layers.

This allows the model to dynamically adapt itself to each individual domain while still benefiting from cross-domain knowledge sharing. Partitioned Normalization (Wang et al., 2019) (PN):A new normalization technique that keeps domain-specific statistics instead of global means used in conventional Batch Normalization. Does not hide domain features by taking average normalization data. Improves accuracy per domain by learning normalization parameters based on the domain identifier.

Star Topology Fully Connected Neural Networks (FCN):
It contains one shared FCN and multiple domain-specific FCNs.
Terminal model parameters per domain are generated by combining shared and domain-specific weights.Enables efficient sharing of parameters without significantly expanding the size of the entire model.Auxiliary Network (Misra et al., 2016).
Directly gets the domain indicator as input and learns the embedding of it.Enhances the domain differentiation ability of the model by having a direct influence on the ultimate prediction.Simple architecture ensures direct impact of domain identity on output result.Resource-Efficient: Reduces the amount of models and parameters required compared to training a model for each domain individually.

Improved Performance: STAR has captured massive gains in real-world applications:

8.0% improvement in CTR (Click-Through Rate)
6.0% improvement in RPM (Revenue Per Mille)

Adaptive Learning: Combines shared learning with domain-specific tuning to improve prediction performance in different business domains.

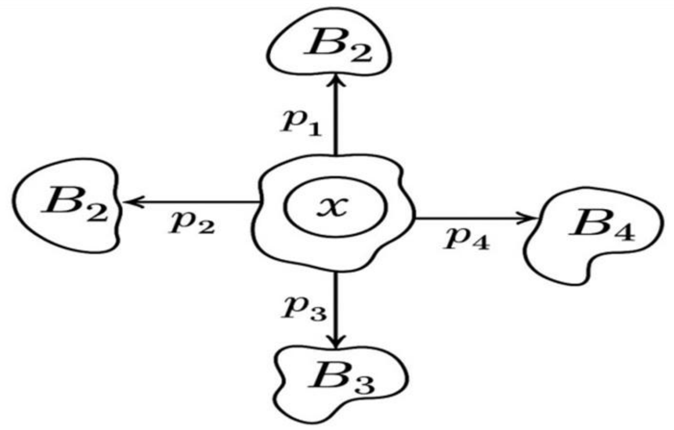
STAR has been successfully implemented in Alibaba's display ad system since late 2020, supporting over 60 business domains. It has shown improved performance compared to previous multi-task and multi-domain learning methods (Ma et al., 2018).[5],[6].

Definition: 1-1 [2]
Suppose that B is a topological space, the fibrewise topology on fibrewise set X over B, mean any topology on X for which the projection P is continuous.

Definition:1-2 [7]
The fibrewise topological space X over B is called fibrewise closed(resp- open) if the projection p is closed (resp- open).

Main Results:

Definition: 2-1 (Fibrewise Star Topological Space)
let $\{B_i\}_{i\in I}$a family of topological spaces, if (X, τ) be a fibrewise topological spaces over $B_{i\in I}$, $i \in I$ with projections p_i for each $i \in I$, then $(X, \tau, B_i\ )_{i\in I}$ is said to be a fibrewise Star topological space over B_i for each $i \in I$ briefly(f,w,S,t,s) .



Example:

Let X be a usual topology on $\mathbb{R}$,B_1 a trivial topology on $\mathbb{R}$,implies X is (f,w,t,s) overB_1 with projection p_1 is any function , if B_2 is lower limit topology, then X is (f,w,t,s) over B_2 with projection identity function, and let B_3 is a countable complement topology, then X is (f,w,t,s) over B_3 ,since X is fibrewise topological space over B_i , i = 1,2,3. Then $(X, \tau, B_i\ )_{i\in I}$ is fibrewise Star topological space over B_i , for each $i \in I$.

Definition:2-3

The fibrewise star topological space $(X, \tau, B_i\ )_{i\in I}$ is called fibrewise star topology closed (resp- open) if the projections p_i is closed (resp- open) functions for each $i \in I$.

Definition: 2-4

A fibrewise star topological space $(X, \tau, B_i\ )_{i\in I}$ is called:

1- **fibrewise star topology closed** if every projection pi:X→Bi is a closed function for all i∈I.
2- **fibrewise star topology open** if every projection pi:X→Bi is an open function for all i∈I.

Lemma:2-5

For a fibrewise star topological space $(X, \tau, B_i\ )_{i\in I}$ over arbitrary base spaces Bi (where $i \in I,\ U_{bi} \subseteq X_{bi}$), the following holds: A subset $U_{bi} \subseteq X_{bi}$ becomes open in Xbi precisely when every point x∈Ubi admits a neighborhood Nx within Xbi satisfying Nx⊆Ubi

Definition: 2-6

Let$(X, \tau, B_i\ )_{i\in I}$ be a f,w,S,t,s and $Ab_i \subseteq P^{-1}(b_i)$, $i \in I$. The subspace of f,w,S,t,s τ_{Abi} on A_{bi} is determined by :

$$\tau_{Ai} = \{U_{bi} \subseteq A : U = W_{bi} \cap A\ for\ som\ W_{bi}\ \subseteq\ P^{-1}(b_i\,)\}$$

Theorem:2-7

If $(X, \tau_1, B_i\ )_{i\in I}$ is a f,w,S,t,s over B_i, $i \in I$, and τ_2 is a topology on X such that $\tau_1\ \subseteq\ \tau_2$. The If $(X, \tau_2, B_i\ )_{i\in I}$ is a f,w,S,t,s over B_i, $i \in I$.

Proof:

Given that all open sets in τ_1 is in τ_2, and for which $(X, \tau_1, B_i\ )_{i\in I}$ is a f,w,S,t,s over B_i for each $i \in I$, then the projections P_i: (X, τ_1) $\longrightarrow B_i$ for each $i \in I$ is a continuos. So P_i: (X, τ_2) $\longrightarrow B_i$ for $i \in I$ is continuos also. Therefor If $(X, \tau_2, B_i\ )_{i\in I}$ is f,w,S,t,s over B_i, $i \in I$.

Definition:2-8

Given a fibrewise star topological space $(X, \tau, B_i)_{i\in I}$ over base spaces Bi (for i∈I), we say a family ω of open subsets of Xbi forms a basis for the fibre Pi−1(bi) when it satisfies the following property: any open set in Xbi can be expressed as a union of members from ω.

Theorem:2-9

Let$(X, \tau, B_i)_{i\in I}$ be a f,w,S,t,s over B_i ,for each $i \in I$,if (x_n) bea a sequence in X converging to x∈ X, then the sequence $(P_i(x_n))$ in B_i, for each $i \in I$ converges to $P_i(x)$, for each $i \in I$.

Consider a fibrewise star topological space $(X, \tau, B_i)_{i\in I}$ over base spaces Bi (indexed by i∈I). Suppose (xn) is a sequence in X converging to x∈X. Then for each projection Pi:X→Bi , the image sequence (Pi(xn)) converges to Pi(x) in Bi, for each i.

Proof:

Since$(X, \tau, B_i)_{i\in I}$ be a f,w,S,t,s over B_i, implies $P_i: X \longrightarrow B_i$ is continuoes projection for each $i \in I$, let V_i be an open subset of B_i for each $i \in I$ containing $P_i(x)$ respectively $i \in I$. Then $P_i^{-1}(V_i)$ is an open subset of X which includes x, one can find N in which $x_n \in P^{-1}(V_i)$ for each $i \in I$, and for all n≥ N. As required.

Conclusion and Future Work:

This paper introduced fibrewise star topological spaces and established foundational results in their theory, expanding the framework of fibrewise topology. Future research directions include investigating compactness and connectedness properties in this setting, as well as exploring potential applications in network systems and computational models. Such extensions could yield deeper insights into the interplay between topology and modern data structures, particularly in multi-domain learning systems like the STAR model. Additionally, examining separation axioms and quotient constructions in fibrewise star spaces may further unify topological and applied perspectives. These advancements could open new avenues for both theoretical development and practical implementations in large-scale systems.

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