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Investigation of even-even $(Ra^{220-224})(Rn^{220-224})(Th^{220-224})$ isobars within the IBM-2 and IVBM

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دراسة النظائر الزوجية–الزوجية $(Ra^{220-224})(Rn^{220-224})(Th^{220-224})$ ضمن إطار نموذج IBM-2 وIVBM

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Abstract:
In this study, we determined the most appropriate limit using the IBM-2 model and the IVBM model that is needed for present calculations of isobars $Ra^{(220-224)}, Rn^{(220-224)},\ and\ Th^{(220-224)}$. To determine the characteristics of each isotope inside the aforementioned isobars, we computed energy levels and compared the outcomes with the experimental data, and then the values of the ratio $\frac{E(I+2)}{E_{2_1^+}}$, the E-GOS curve, and the ratio $\frac{r(I+2)}{I}$ were adopted. From these values it was possible to determine the dynamic symmetries of each isotope, since (Ra^{222}, Rn^{224}) were classified as having O(6) symmetry, while the isobars (Ra^{224}, Th^{224}) have X(5) symmetry, the isobars (Ra^{220}, Rn^{220}) and (Rn^{222}, Th^{222}) have E(5) symmetry, and the Th^{220} has U(5) symmetry. The phenomenon of back bending of these isobars, which was all unbinding, was studied.
Analysis was done on the startling energy differential between the negative party band states and the ground state band.

Keywords: Interacting vector boson model, Odd- Even Staggering, back bending, Isobars, Energy ratio, E-GOS.

الملخص
في هذه الدراسة، تم تحديد التحديد الأكثر ملاءمة باستخدام نموذج **IBM-2** ونموذج **IVBM** اللازم للحسابات الحالية للنظائر $Ra^{(220-224)}, Rn^{(220-224)},\ and\ Th^{(220-224)}$ ولتحديد الخصائص المميزة لكل نظير ضمن النظائر المذكورة، قمنا بحساب مستويات الطاقة ومقارنة النتائج بالبيانات التجريبية. ثم اعتماد قيم النسبة $\frac{E(I+2)}{E_{2_1^+}}$ ومنحنى E-GOS والنسبة $\frac{r(I+2)}{I}$

ومن خلال هذه القيم، كان من الممكن تحديد التناظرات الديناميكية لكل نظير؛ إذ تم تصنيف (Ra^{222}, Rn^{224})على أنها ذات تناظرO(6)، في حين أن النظائر (Ra^{224}, Th^{224}) تمتلك تناظر X(5)، والنظائر (Ra^{220}, Rn^{220}) وكذلك (Rn^{222}, Th^{222}) تمتلك تناظر E(5)، أما Th^{220} فله تناظر U(5).
كما تم دراسة ظاهرة الانحناء الخلفي (Back Bending)في هذه النظائر، والتي كانت جميعها unbinding. وتم إجراء تحليل للتأرجح الطاقي اللافت بين حالات حزمة الحزم السالبة وحزمة الحالة الأرضية.

الكلمات المفتاحية: نموذج البوزونات المتجهة المتفاعلة، التأرجح الزوجي–الفردي(Odd–Even Staggering) ، الانحناء الخلفي، النظائر، نسبة الطاقة، E-GOS.

1. Introduction:

Spectroscopy of medium mass and heavy (even–even) nuclei and the study of collective nuclear motion are some of the most interesting topics in nuclear physics [1]. Numerous models have been developed to investigate various nuclei due to the lack of a consistent theory that can be used to their study [2]. Two important phenomenological techniques that effectively characterize nuclear collective motion in terms of bosons are represented by IBM-2 and IVBM inside these models. The interacting boson model (IBM-2), initially introduced by Arima and Iachello, is one of the algebraic models for studying nuclear structure. [3]. Based on the group theoretical approach, this model defines different kinds of nuclear collective states in (even-even) nuclei. The inner core of the (even–even) nucleus is thought to be joined with bosons, which represent pairs of identical valence nucleons with angular momenta of (J = 0) for the s state or (J= 2) for the d state which are treated as distinct constituentsThe proton p and neutron n bosons, which make up the collective excitations in the nucleus when the vector bosons utilized in it have angular momentum (J=1) [4], are the basis of the interacting vector boson model (IVBM), which was created by Ganev et al. [5]. These algebraic Hamiltonian-based models, which are ideal for studying phase transitions, have been widely used to study nuclei from various mass regions. These models' nuclear forms, where the transition occurs, are linked to dynamic symmetries, which allow for the extraction of analytical solutions for the pertinent observables. [6]. In the IBM-2 model is possible to reproduce the properties of three different classes of nuclei which in the conventional nomenclature correspond to the vibrational, axially symmetric deformed rotor and the γ-unstable nuclei rotor is related to subgroups, beginning with the top group U (6), which is known by the labels U (5), O (6), and SU (3) [7], [8], and [9]. As a result, most collectively structured nuclei exhibit a variety of features rather than just one symmetry, It was suggested that a new class of symmetries be used for systems that are localized at critical locations. It has recently been suggested that nuclei at the locations of phase transitions between various dynamical symmetries can be described by the critical-point symmetries E (5) and X (5). The E (5) critical point symmetry represents the phase transition between U(5) and O(6), while the X (5) critical point symmetry describes the phase transition between U(5) and SU (3) [10], [11]. We have investigated isobars' collective characteristics (($Ra^{(220-224)}$, $Rn^{(220-224)}$, $and\ Th^{(220-224)}$)) in the framework of IBM-2 and IVBM models then contrasted the outcomes with the values from the trial, in sec. 2 and 3.

2. How the IBM-2 Hamiltonian is constructed:

One way to express the IBM-2 Hamiltonian is[12], [13]:

$$\hat{H} = \hat{H}_\pi + \hat{H}_\nu + \hat{H}_{\pi\nu} \qquad (1)$$

Here, the operator $\hat{H}_\rho$ with ($\rho = \pi, \nu$) includes the single ρ boson energies and the two-body interaction among them. The operator $\hat{H}_\rho$ can be written as;

$$\hat{H}_\rho = \varepsilon_\rho \hat{n}_{d_\rho} + \hat{V}_{\rho\rho} \qquad (\rho = \pi, \nu) \qquad (2)$$

ε_ρ is the binding energy of the d bosons of the ρ boson, and the operator n ̂_(d_ρ) counts the number of d bosons of the ρ boson.

ε_ρ is the energy of the d bosons of ρ boson, the operator $\hat{n}_{d_\rho}$is a counter to count the number of d bosons of ρ boson, and the operator $\hat{V}_{\rho\rho}$ is the interaction of identical bosons.

3. Building the IVBM Hamiltonian:

The IVBM (Interacting Vector Boson Model) was first proposed by Ganev et al. to determine the energy levels of the ground state band (GSB) and negative parity band (NPB) of even-even nuclei. In the IVBM model, the interaction between the protons' and neutrons' vector bosons is considered independently [14], [9]. This model's Hamiltonian can be expressed as follows:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2 \ldots\ldots\ldots\ldots.\ldots (3)$$

In this instance, the model's parameters a, b, and β_3 define the ground state band, while $\alpha_1 and\ \alpha_3$ describes the octupole band. Although T^2 and T_0 represent the pseudospin (the quantum number used to differentiate between two kinds of vector bosons), which are the essential components of the model's algebraic structure. The total number of bosons is represented by the Hermitian operator, N.

The permitted values for the two bands, GSB and NPB energy states, in the (IVBM) model are given by:

$$E(I)_{GSB} = \beta I(I+1) + \gamma I \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and

$$E(I)_{NPB} = \beta I(I+1) + (\gamma + \eta)I + \zeta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The nuclei are highly affected by the vibrational properties as indicated by the γ parameter, and by the rotational properties as indicated by the β parameter. To ascertain the values of the energy levels in the NPB beam, the parameters η and ζ are essential additions [8], [15], [16].

4. Nuclear shape transition:

Nuclear shape phases reflect the collective motion modes of nuclei [17], numerous techniques have been developed investigate and pinpoint these symmetries.

4.1. Energy ratio $R_{4/2}$.

One of the best indicators of shape transition along isotopic chains is the behavior ratio of the second to the first excited states of various nuclei. This ratio ranges from the value that corresponds to vibrators around a spherical shape ($R_{4/2} = 2$) to the excitation of the deformed rotor ($R_{4/2} = 3.33$) and ($R_{4/2} = 2.5$) for γ-unstable [18], [19].

4.2. EGOS curves.

The relation known as energy gamma over spin (E-GOS) curves, which was introduced by Regan et al. [16], allows vibrational and rotational behavior, as well as the transition between the two, to be vividly highlighted without requiring any structural preconceptions. This relationship offers important insights into the evolution of (even-even) nuclei in the Yrast band. For three limits, these are the relations[18], [20], [21], [22]:

$$(Vibrational)\qquad R(I) = \frac{\hbar\omega}{I} \xrightarrow{I\to\infty} 0 \qquad (6)$$

$$(Rotational)\qquad R(I) = \frac{\hbar^2}{2J}\left(4 - \frac{2}{I}\right) \xrightarrow{I\to\infty} 4\left(\frac{\hbar^2}{2J}\right) \qquad (7)$$

$$(\gamma - soft)\qquad R(I) = \frac{E_{2_1^+}}{4}\left(1 + \frac{2}{I}\right) \xrightarrow{I\to\infty} \frac{E_{2_1^+}}{4} \qquad (8)$$

4.3. The ratio $r\left(\frac{I+2}{I}\right)$.

The symmetry for the excited band of (even − even) nuclei was defined by constructing the following ratios for a given for each spin.

$$r\left(\frac{I+2}{I}\right) = \left[R\left(\frac{I+2}{I}\right)_{exp} - \frac{I+2}{I}\right] \times \frac{I(I+1)}{2(I+2)} \qquad (9)$$

Where $R(\frac{I+2}{I})_{exp}$ is the ratio of the experimental value [18], the characteristics of each nucleus were determined, when this relationship is applied to a group of distinct nuclei, the limits are as the follows: $0.1 \le r \le 0.35$ for vibrational nuclei, $0.4 \le r \le 0.6$ for $\gamma - unstable$ nuclei, and $0.6 \le r \le 1$ for rotating nuclei.

5. Odd- Even Staggering (OES).

The relative displacement of the odd angular momentum levels with respect to their adjacent levels with even angular momentum is known as (odd-even) staggering [23]. The following formula can be used to determine the (odd-even) staggering:

$$\Delta E_{1,\gamma}(I) = {}^{1}\!/_{16}\left[6E_{1,\gamma}(I) - 4E_{1,\gamma}(I-1) - 4E_{1,\gamma}(I+1) + E_{1,\gamma}(I-2) + E_{1,\gamma}(I+2)\right] \qquad (10)$$

Where $E_{1,\gamma}(I) = E(I+1) - E(I)$, $\Delta E_{1,\gamma}(I)$ displays alternate sign values over the extended angular momentum area [18].

6. The *back bending* phenomena.

Johnson A., et al. have found this phenomenon [24]. They discovered that while the moment of inertia at a given angular momentum greatly rises, even while the rotational energy value of the γ-transition from a state with momentum I to a state with momentum I-2 of particular nuclei reduces [25]. This behavior causes the energy $\hbar\omega$ value [2] to bend backward or upward, as defined by:

$$\hbar\omega = \frac{\Delta E}{\sqrt{I(I+1)} - \sqrt{(I-2)(I-1)}} \qquad (11)$$

The following provides the moment of inertia:

$$\frac{2\vartheta}{\hbar^2} = \frac{4I-2}{\Delta E} \qquad (12)$$

Where $\Delta E = E(I) - E(I-2)$.

7. Outcomes and discussion:

The IBM-2 Hamiltonian parameters used in this study to calculate the energies of the positive parity low-lying levels of $\boldsymbol{Ra^{220-224}}, \boldsymbol{Rn^{220-224}}, \boldsymbol{and\ Th^{220-224}}$ are listed in Table 1. The values of the IBM-2 Hamiltonian parameters were found by fitting the experimental energy levels and letting one parameter change while maintaining the others constant. This process was continued until an overall match was achieved. The IVBM settings are also mentioned in that table.

For more information about isobars nuclei under-study, we tested the presence of back bending on it, and as shown in Fig. 5, all were unbending with some different skews in the curve for the ${}^{220}_{90}Th_{130}$ isotope. Since, I. M. Ahmed, et al. (2018), show results that are somewhat similar for Thorium isotopes in ref. [2].

Table1. The parameters used for IBM-2 and IVBM calculations, all parameters are given in MeV.

		IBM-2 parameters							IVBM parameters			
Nuclei	**$N\pi$ +$N\upsilon$ =N**	ε_d	$\kappa_{\pi\upsilon}$	χ_π	χ_ν	$\xi_{1,2,3}$	$C_{0,2,4}L_\pi$	$C_{0,2,4}L_\upsilon$	β	γ	η	ζ
^{220}Rn	**2+4 =6**	**0.398**	**-0.048**	**-1.322**	**-1.322**	**0.000 -0.178 0.000**	**0.000 0.000 0.000**	**0.000 0.000 0.000**	**0.0046**	**0.1068**	**-0.0296**	**0.3762**
^{222}Rn	**2 +5=7**	**0.375**	**-0.050**	**-1.322**	**-1.322**	**0.000 -0.178 0.000**	**0.000 0.010 0.000**	**0.000 0.000 0.000**	**0.0055**	**0.0765**	**-0.0354**	**0.5487**
^{224}Rn	**2+6= 8**	**0.318**	**-0.065**	**-1.322**	**-1.322**	**0.000 -0.178 0.000**	**0.025 0.200 0.000**	**0.058 0.000 0.053**	**0.0068**	**0.0473**	**-0.0321**	**0.5234**
^{220}Ra	**3+3 =6**	**0.520**	**-0.046**	**-1.322**	**-1.322**	**0.200 -0.178 0.000**	**0.070 0.200 0.000**	**0.100 0.000 -0.040**	**0.0039**	**0.0777**	**-0.0116**	**0.3391**
^{222}Ra	**3 +4=7**	**0.478**	**-0.045**	**-1.322**	**-1.322**	**0.000 -0.178 0.000**	**0.060 0.200 0.00**	**0.200 0.000 -0.030**	**0.006**	**0.0374**	**-0.0079**	**0.2007**
^{224}Ra	**3+5= 8**	**0.483**	**-0.047**	**-1.322**	**-1.322**	**0.000 -0.178 0.000**	**0.030 0.200 0.000**	**0.200 0.100 -0.028**	**0.0062**	**0.0237**	**-0.0089**	**0.1895**
^{220}Th	**4+2 =6**	**0.945**	**-0.089**	**5.740**	**2.460**	**1.780 -0.165 1.504**	**0.810 0.470 -0.110**	**0.410 0.420 -0.320**	**0.0011**	**0.1901**	**-0.0054**	**0.0374**
^{222}Th	**4 +3=7**	**0.977**	**-0.089**	**5.320**	**2.529**	**0.970 -0.130 1.090**	**0.940 0.372 -0.152**	**0.680 0.120 -0.310**	**0.0047**	**0.0774**	**-0.0060**	**0.1652**
^{224}Th	**4+4= 8**	**0.979**	**-0.089**	**5.330**	**2.450**	**0.950 -0.130 1.090**	**0.920 0.311 -0.137**	**0.690 -0.001 -0.295**	**0.0069**	**0.0284**	**-0.0114**	**0.2202**

Using those parameters, we calculated the energy levels and they were very close to the experimental ones [14]. We performed several tests to determine the structural properties of the nuclei under investigation. The first test was the ratio test $\left(R_{4/2}\right)$, and Figure 1 shows the results for doing $\left(R_{(I+2)/2}\right)$ test. Next, as illustrated in Figure 2, we conducted the E-GOS test. Next, as illustrated in Figure 3, we conducted the ratio $r\left(\frac{I+2}{I}\right)$ test.

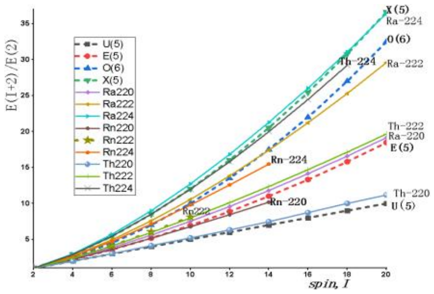

Fig. 1 (energy ratio test).

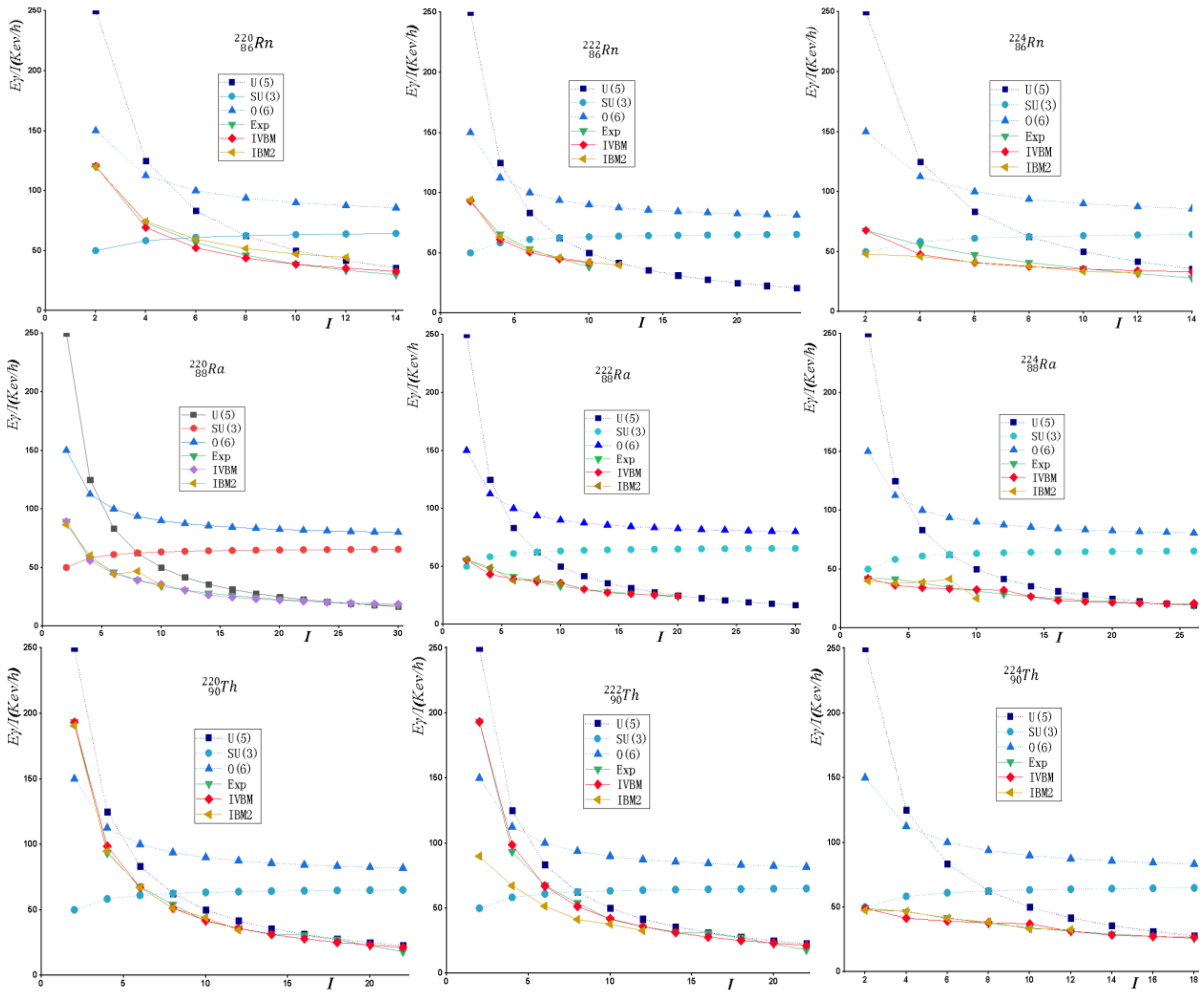

Fig.2 (E-GOS test).

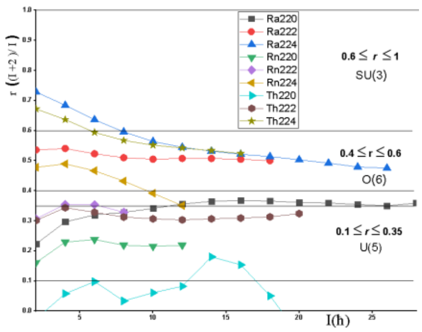

Fig. 3 (ratio r test).

We were able to determine from these tests that the $^{220}_{90}Th_{130}$, isotope possesses vibrational symmetry U(5) since $^{220}_{88}Ra_{132}$, $^{220}_{86}Rn_{134}$, $^{222}_{86}Rn_{136}$, *and* $^{222}_{90}Th_{132}$ have E(5) symmetry, the $^{222}_{88}Ra_{134}$ *and* $^{224}_{86}Rn_{138}$ *have* gamma soft symmetry O(6), and $^{224}_{88}Ra_{136}$, $^{224}_{90}Th_{134}$ *isobars* possess X(5) symmetry. Table 2 provides a summary of these findings.

Table 2: Isobars ($^{220-224}_{86}Rn$, $^{220-224}_{88}Ra$, $^{220-224}_{90}Th$) classification**.**

X (5)	O (6)	E (5)	U (5)
$^{224}_{88}Ra_{136}$	$^{222}_{88}Ra_{134}$	$^{220}_{88}Ra_{132}$	$^{220}_{90}Th_{130}$
$^{224}_{90}Th_{134}$	$^{224}_{86}Rn_{138}$	$^{220}_{86}Rn_{134}$	
		$^{222}_{86}Rn_{136}$	
		$^{222}_{90}Th_{132}$	

The apparent staggering in the disparities between the isomers' GSB and NPB energies is depicted in Fig. 4. The amazing did not approach zero except in the $^{220}_{90}Th_{130}$ isotope, which indicates that this isotope is stable and non-deformable.

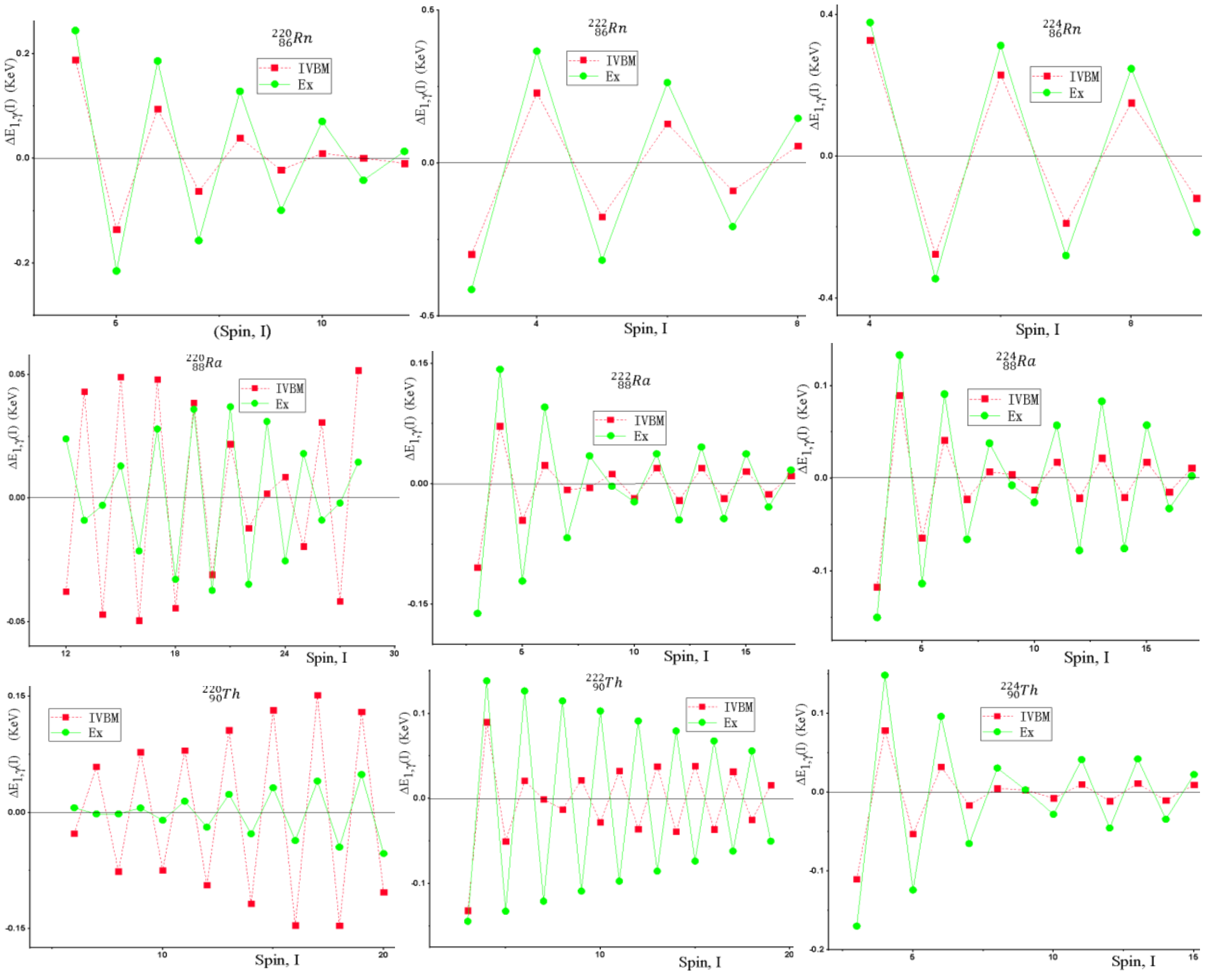

Fig. 4 (staggering test).

For more information about isobars nuclei under-study, we tested the presence of back bending on it, and as shown in Fig. 5, all were unbending with some different skews in the curve for the $^{220}_{90}Th_{130}$ isotope. Since, I. M. Ahmed, et al. (2018), show results that are somewhat similar for Thorium isotopes in ref. [2].

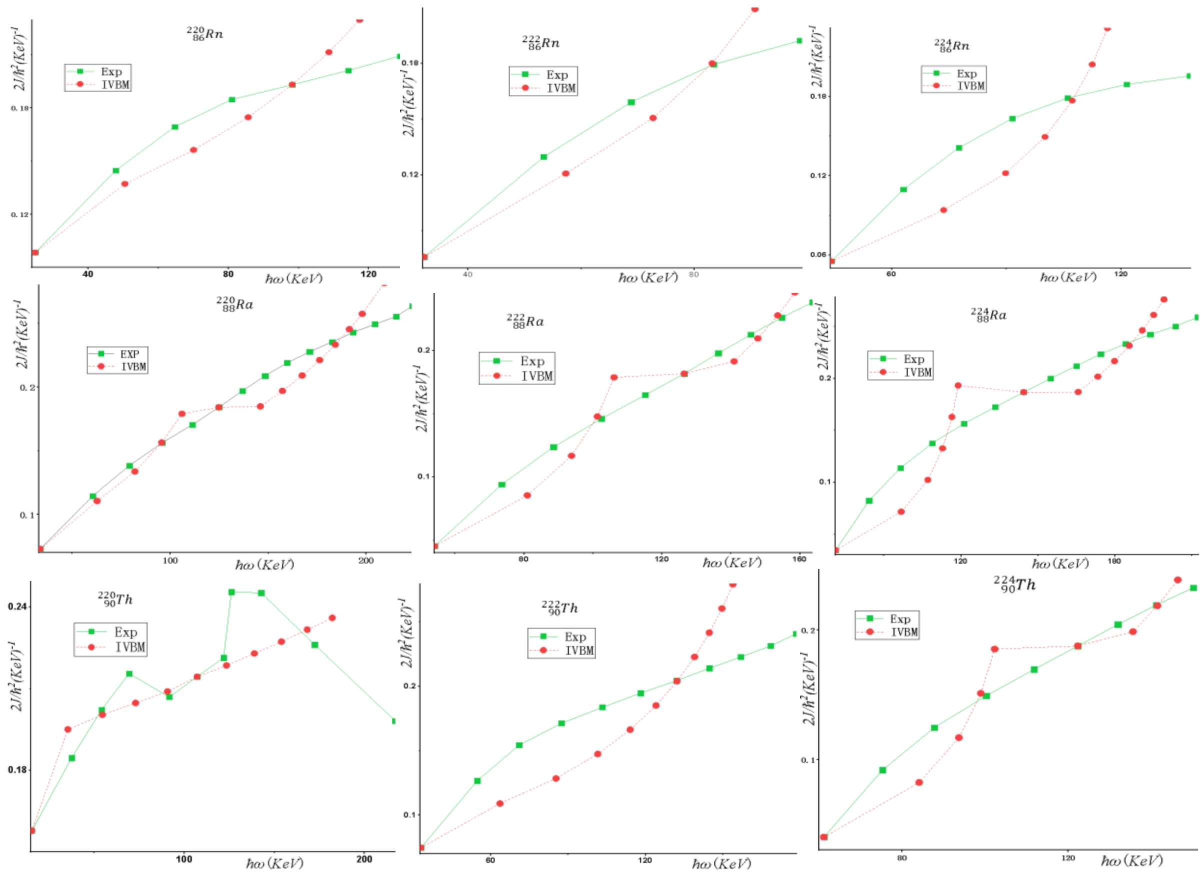

Fig. 5 (back bending test).

8. Conclusion: To sum up, IBM-2 uses the NPBOS software and IVBM with MATLAB to calculate the positive parity energy levels for ($Rn^{220-224}$) isotopes with neutron numbers 134 to 138, ($Ra^{220-224}$) isotopes with neutron numbers 132 to 136, and ($Th^{220-224}$) isotopes with neutron numbers 130 to 134. However, the sole technique utilized to ascertain the negative parity energy levels was IVBM. The investigation shows that there is a good agreement between the results of these models and the available experimental data. The ratios $\left(R_{4/2}\right)$ and $r\left(\frac{I+2}{I}\right)$ have been used to characterize the GSB of these isobars. Additionally, the ideal limits of gamma-soft, vibrational, and rotational scenarios were plotted against the energy gamma over spin (E-GOS) curves of the GSB for the previously stated nuclei. The staggered test

The staggering test has been done also where, no under-study isotope has stability on zero, except $^{220}_{90}Th$. Finally, the back bending test was also performed, revealing a clear unbending in all isotopes except $^{220}_{90}Th$, where a slight back bending was observed.

The findings of this study demonstrate that the IVBM clarifies strong agreement with the experimental data and offers a better explanation of the excitation energies of the Yrast-band of even-even isobars nuclei than the IBM-2. The shape transition for these isobars is from U (5) toward X (5), according to the results obtained for $E_{I+2}/E_{2_1^+}$values. The presence of the E-GOS curve and the ratio (r(I+2)/I) confirm that the isobars under study are located in the mentioned region.

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