African Journal of Advanced Pure and Applied Sciences (AJAPAS)
Online ISSN: 2957-644X
Volume 5, Issue 2, 2026
Page No: 340-347
Website: https://aaasjournals.com/index.php/ajapas/index
ISI 2025: 1.126 SJIFactor 2024: 6.752 معامل التأثير العربي: 1.62

Analytical Solutions of the (2+1)-Dimensional Stochastic Chiral Nonlinear Schrödinger equation Using the Generalized $(G'/G+A)$-Expansion and Jacobi Elliptic Function Methods

Muna Ali Mansour [1 *], Fatma M. Ahmed [2], Mahmoud El-Horbaty [3]
[1, 2] Mathematics Department, Faculty of Sciences, Zawia University, Alajelat, Libya
[3] Mathematics Department, Faculty of Sciences, Zagazig University, Zagazig, Egypt

الحلول التحليلية لمعادلة شرودنجر اللاخطية الكيرالية العشوائية ذات الأبعاد (1+2) باستخدام طريقتي التوسيع المعمم $(G'/G+A)$ ودوال جاكوبي الإهليجية

منى علي منصور [1*]، فاطمة مفتاح أحمد[2]، محمود إبراهيم الهيربيطي[3]
[1، 2] قسم الرياضيات، كلية العلوم العجيلات، جامعة الزاوية، االعجيلات، ليبيا
[3] قسم الرياضيات، كلية العلوم، جامعة الزقازيق، الزقازيق، مصر

[*]Corresponding author: m.mansour@zu.edu.ly

Received: March 23, 2026	Accepted: June 06, 2026	Published: June 17, 2026

Abstract
This study focuses on the (2+1)-dimensional stochastic Chiral Nonlinear Schrödinger equation, which incorporates a multiplicative Gaussian noise component. By applying the generalized (G′⁄G+A)-expansion method and the Jacobi elliptic function method, various accurate analytical solutions were derived, including bright solitons, Kink-type dark solitons, and periodic solitons. The research also evaluates the physical stability and structural integrity of these waves under the influence of random disturbances. Numerical simulations showed that the solutions obtained maintain their properties despite the presence of noise, which confirms the robustness of the analytical results. These results provide important insights into wave propagation in random nonlinear media, particularly in the fields of optical fibers and plasma physics.

Keywords: (2+1)-dimensional stochastic Chiral Nonlinear Schrödinger equation, Generalized (G′⁄G+A)-expansion method, Jacobi elliptic function method, Multiplicative Gaussian noise, Optical solitons.

الملخص
تركز هذه الدراسة على معادلة شرودنغر العشوائية اللاخطية ذات الأبعاد (1+2) ، والتي تتضمن عنصر ضوضاء غاوسي مضاعف. من خلال تطبيق طريقة التمدد المعممة (G′⁄G+A) وطريقة دالة جاكوبي الإهليلجية، تم استخلاص العديد من الحلول التحليلية الدقيقة، بما في ذلك السوليتونات الساطعة، والسوليتونات المظلمة من نوع Kink، والسوليتونات الدورية. كما قام البحث بتقييم ثبات الاستقرار الفيزيائي والسلامة الهيكلية لهذه الموجات تحت تأثير الاضطرابات العشوائية. وأظهرت عمليات المحاكاة العددية أن الحلول التي تم الحصول عليها تحافظ على خواصها بالرغم من وجود الضوضاء مما يؤكد متانة النتائج التحليلية. توفر هذه النتائج رؤى مهمة حول انتشار الموجات في الوسائط العشوائية غير الخطية، خاصة في مجالات الألياف الضوئية وفيزياء البلازما.

الكلمات المفتاحية: معادلة شرودنجر العشوائية اللاخطية ذات الأبعاد (1+2)، طريقة التمدد المعممة (G′⁄G+A)، طريقة دالة جاكوبي الإهليلجية، الضوضاء الغوسية المضاعفة، السوليتونات الضوئية.

Introduction

Nonlinear evolution equations are essential for comprehending intricate natural phenomena in areas such as optics and plasma physics. One key example is the Chiral nonlinear Schrödinger equation (CNLSE), which is widely used to study wave propagation in quantum devices [1, 12]. Recently, there has been growing interest in stochastic generalizations of these models to provide a more realistic description of wave dynamics in optical and plasma systems [8, 11]. Two effective techniques for obtaining exact solutions are the (G′⁄G+A)-expansion method [5, 6, 7] and the Jacobi elliptic function method [1]. For example, El-Horbaty et al. investigated optical solitons in birefringent fibers [8], and Albosaily et al. studied the exact solutions of the two-dimensional stochastic Chiral nonlinear Schrödinger equation (2D-SCNLSE) under the influence of multiplicative noise [9, 10].

In this study, we focus on the (2+1)-dimensional stochastic CNLSE with multiplicative Gaussian noise. We employ the generalized (G′⁄G+A)-expansion method and the Jacobi elliptic function method to derive exact analytical solutions. By explicitly incorporating the system's stochastic nature into the phase of the wave envelope, we can analyze the impact of noise on the amplitude, phase, and stability of solitary and periodic waves. We also perform numerical simulations to illustrate the impact of noise on the obtained solutions and validate the robustness of these analytical results under stochastic perturbations. This study provides a comprehensive framework for investigating multidimensional stochastic nonlinear wave propagation, extending the solution space of CNLSEs and serving as a reference for future theoretical and experimental investigations in optical and plasma systems.

1.1 The Governing Model,

The (2 + 1)-dimensional stochastic cubic nonlinear Schrödinger equation (2D-SCNLSE) with multiplicative white noise is given by:

$$i\psi_t + a\left(\psi_{xx} + \psi_{yy}\right) + i\left[b_1(\psi\psi_x^* - \psi^*\psi_x) + b_2\left(\psi\psi_y^* - \psi^*\psi_y\right)\right]\psi + \sigma\psi\dot{\beta}(t) = 0, \quad (1)$$

where $\psi(x, y, t)$ is a complex wave function, a is the dispersion coefficient, b_1 and b_2 are real nonlinearity parameters, σ is the noise intensity, $\beta(t)$ is standard Brownian motion, and $\dot{\beta}(t)$ denotes white noise.

This model is widely used to describe wave propagation in nonlinear media subject to random environmental fluctuations, such as those in optical fibers and plasma physics.

Following the reduction steps in [9], Eq. (1) is transformed into a stochastic ordinary differential equation (ODE). In this work, we investigate the exact solutions of this model by employing two different analytical methods to provide a comprehensive comparative study.

1.2. Mathematical Preliminaries,

To derive exact analytical solutions, we employ the travelling wave transformation

$$\psi(x, y, t) = u(\xi)e^{i\theta}, \qquad \xi = \rho_1 x + \rho_2 y + \rho_3 t, \qquad \theta = \theta_1 x + \theta_2 y + \theta_3 t, \quad (2)$$

where $\rho_1,\ \rho_2,\ \rho_3, \theta_1, \theta_2,\ and\ \theta_3$ are nonzero real constants and $u(\xi)$ is a real-valued function.

Substituting transformation (2) into Eq. (1) and separating the real and imaginary parts yields the following equations. From the imaginary part, we obtain

$$\rho_3 = -(2a\rho_1\theta_1 + 2a\rho_2\theta_2), \quad (3)$$

From the real part, the governing equation is reduced to the nonlinear ordinary differential equation:

$$u'' - \mathrm{P}_1 u^3 - \mathrm{P}_2 u = 0, \quad (4)$$

where the parameters P_1 and P_2 are defined as

$$\mathrm{P}_1 = \frac{2(b_1\theta_1 + b_2\theta_2)}{a(\rho_1^2 + \rho_2^2)}, \qquad \mathrm{P}_1 = \frac{\theta_3 + a(\theta_1^2 + \theta_2^2)}{a(\rho_1^2 + \rho_2^2)}. \quad (5)$$

In the following sections, two analytical techniques are applied to Eq. (4) to construct exact solitary and periodic wave solutions of the stochastic CNLSE.

2. The $(G'/G + A)$ - Expansion Method

To that end, using a balance between u'' with u^3 in Eq. (4), we obtain $N = 1.$ According to the scheme of this method [5, 6, 7, 10], we assume the formal solution of Eq. (6) to be

$$u(\xi) = a_0 + a_1\left(\frac{G'(\xi)}{G(\xi)} + A\right), \quad (6)$$

where a_0 , a_1, and A are constants to be determined, such as $a_1 \neq 0$, and $G(\xi)$ satisfies

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \quad (7)$$

where λ and μ are constants. It is known [8] that (G'/G) takes the following form. The $(G'/G + A)$ method uses the same steps:

$$\frac{G\prime}{G} = \begin{cases} A - \frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\,\frac{C_1 e^{\frac{\sqrt{\Delta}}{2}\xi} - C_2 e^{-\frac{\sqrt{\Delta}}{2}\xi}}{C_1 e^{\frac{\sqrt{\Delta}}{2}\xi} + C_2 e^{-\frac{\sqrt{\Delta}}{2}\xi}}, & if\ \Delta > 0 \\ A - \frac{\lambda}{2} + \frac{\sqrt{-\Delta}}{2} tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right) & if\ \Delta < 0 \\ A - \frac{\lambda}{2} + \frac{C_2}{C_1 + C_2\xi} & if\ \Delta = 0 \end{cases}, \tag{8}$$

where C_1, C_2 are arbitrary constants, while $\Delta = \lambda^2 - 4\mu$. Putting Eq. (5) along with Eq. (6) into Eq. (4), collecting all the coefficients of powers of $\left[\frac{G\prime}{G}\right]^i$, $(i = 0, 1, 2, 3)$ and setting them to zero, we obtain:

$$\left.\begin{aligned} &[G'/G]^3:\ 2a_1 - \mathrm{P}_1 a_1^3 = 0 \\ &[G'/G]^2:\ 3a_1(\lambda - 2A) - 3\mathrm{P}_1 a_0 a_1^2 = 0 \\ &[G'/G]^1:\ a_1[(\lambda - 2A)^2 + 2(\mu - A\lambda + A^2)] - 3\mathrm{P}_1 a_0^2 a_1 - \mathrm{P}_2 a_1 = 0 \\ &[G'/G]^0:\ a_1(\lambda - 2A)(\mu - A\lambda + A^2) - \mathrm{P}_1 a_0^3 - \mathrm{P}_2 a_0 = 0 \end{aligned}\right\}, \tag{9}$$

By handing the system in Eq. (9) gives rise to

$$a_1 = \pm\sqrt{\frac{2}{\mathrm{P}_1}}, \quad a_0 = -a_1\left(A + \frac{\lambda}{2}\right), \tag{10}$$

From (5) and (10), we obtain the general solution:

$$u(\xi) = a_1\left(\frac{G'(\xi)}{G(\xi)} - \frac{\lambda}{2}\right), \tag{11}$$

And the frequencies of solutions

$$\mathrm{P}_2 = \frac{\lambda^2 - 4\mu}{2}, \tag{12}$$

Now, the solution of Eq. (4) becomes

$$u(\xi) = \pm\sqrt{\frac{2}{\mathrm{P}_1}}\left(\frac{G'(\xi)}{G(\xi)} - \frac{\lambda}{2}\right), \tag{13}$$

Case 1: If $\Delta > 0$, we obtain the following six hyperbolic solutions:

$$u_1(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}} tanh\left(\frac{\Delta}{2}\xi\right), \tag{14}$$

Where $C_1 \neq 0$ and $C_2 = 0$.

$$u_2(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}} coth\left(\frac{\Delta}{2}\xi\right), \tag{15}$$

Where $C_1 = 0$ and $C_2 \neq 0$.

$$u_3(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}}\left[tanh\left(\frac{\Delta}{2}\xi + tanh^{-1}\left(\frac{C_2}{C_1}\right)\right)\right], \tag{16}$$

Where $C_1 \neq 0$ and $C_2 = 0$, and $C_1^2 < C_2^2$.

$$u_4(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}}\left[coth\left(\frac{\Delta}{2}\xi + coth^{-1}\left(\frac{C_1}{C_2}\right)\right)\right], \tag{17}$$

Where $C_1 = 0$ and $C_2 \neq 0$, and $C_1^2 > C_2^2$.

$$u_5(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}}\left[1 + tanh\left(\frac{\Delta}{2}\xi\right)\right], \tag{18}$$

Where $C_1 = C_2$.

$$u_6(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}}\left[1 - coth\left(\frac{\Delta}{2}\xi\right)\right], \tag{19}$$

Where $C_1 = -C_2$.

Case 2: If $\Delta < 0$, we obtain the following six periodic solutions:

$$u_7(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}} tan\left(\frac{-\Delta}{2}\xi\right), \tag{20}$$

Where $C_1 \neq 0$ and $C_2 = 0$.

$$u_8(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}} cot\left(\frac{-\Delta}{2}\xi\right), \tag{21}$$

Where $C_1 = 0$ and $C_2 \neq 0$.

$$u_9(\xi) = \pm\frac{\Delta}{\sqrt{2\mathrm{P}_1}}\left[tan\left(\frac{-\Delta}{2}\xi + tan^{-1}\left(\frac{C_2}{C_1}\right)\right)\right], \tag{22}$$

Where $C_1 \neq 0$ and $C_2 = 0$, and $C_1^2 < C_2^2$.

$$u_{10}(\xi) = \pm \frac{\Delta}{\sqrt{2P_1}}\left[cot\left(\frac{-\Delta}{2}\xi + cot^{-1}\left(\frac{C_1}{C_2}\right)\right)\right], \tag{23}$$

Where $C_1 = 0$ and $C_2 \neq 0$, and $C_1^2 > C_2^2$.

$$u_{11}(\xi) = \pm \frac{\Delta}{2\sqrt{2P_1}}\left[tan\left(\frac{-\Delta}{2}\xi\right) - cot\left(\frac{-\Delta}{2}\xi\right)\right], \tag{24}$$

Where $C_1 = C_2$.

$$u_{12}(\xi) = \pm \frac{\Delta}{2\sqrt{2P_1}}\left[tan\left(\frac{-\Delta}{2}\xi\right) + cot\left(\frac{-\Delta}{2}\xi\right)\right], \tag{25}$$

Where $C_1 = -C_2$.

Case 3: If $\Delta = 0$, we obtain the following two solutions:

$$u_{13}(\xi) = \pm\sqrt{\frac{2}{P_1}}\left(\frac{1}{\xi}\right), \tag{26}$$

Where $C_1 \neq 0$ and $C_2 = 0$.

$$u_{14}(\xi) = \pm\sqrt{\frac{2}{P_1}}\left(\frac{C_2}{C_1 + C_2\xi}\right), \tag{27}$$

Where $C_1 \neq 0$ and $C_2 \neq 0$.

The solutions Eq. (14) – Eq. (19) exist under the constraints condition $(\lambda^2 - 4\mu)P_1 < 0$ for hyperbolic solutions, and $(4\mu - \lambda^2)P_1 < 0$ for trigonometric solutions.

3. The Jacobi Elliptic Function Expansion Method

According to the Jacobi elliptic expansion approach [1], the formal solution of Eq. (4) is assumed to be:

$$u(\xi) = c_0 + c_1 Y(\xi), \tag{28}$$

where c_0 and c_1 are constants to be determined, and the function $Y(\xi)$ satisfies the Jacobi elliptic equation:

$$(Y')^2 = \alpha Y^4 + \beta Y^2 + \gamma, \tag{29}$$

where α, β and γ are fixed constants depending on the modulus $(0 < n < 1)$.

Differentiating Eq. (29) gives:

$$Y'' = 2\alpha Y^3 + \beta Y \tag{30}$$

Substituting Eq. (28) into Eq. (4) and setting $c_0 = 0$, this is a deliberate choice with a physical motive, not a random choice, and thus we compare the coefficients of Y^3 and Y:

Y^3: $2\alpha c_0 - P_1 c_1^3 = 0 \Rightarrow c_1 = \sqrt{\frac{2\alpha}{P_1}}$.

Y: $\beta c_1 - P_2 c_1 = 0 \Rightarrow \beta = P_2$.

The parameters α, β and γ are determined by the specific properties of the chosen Jacobi elliptic functions as follows:

Case 1: $sn(\xi, n)$ solution

Taking $\alpha = n^2k^2, \beta = -(1 + n^2)k^2$, and $\gamma = k^2$, the solution is:

$$\psi_1(x, y, t) = \left[\sqrt{\frac{2n^2k^2}{P_1}} sn(k\xi, n)\right] e^{i\theta}, \tag{31}$$

provided $P_2 = -(1 + n^2)k^2$.

Case 2: $cn(\xi, n)$ solution

Taking $\alpha = -n^2k^2, \beta = (-1 + 2n^2)k^2$, and $\gamma = (1 - n^2)k^2$, the solution is:

$$\psi_2(x, y, t) = \left[\sqrt{\frac{-2n^2k^2}{P_1}} cn(k\xi, n)\right] e^{i\theta}, \tag{32}$$

provided $P_2 = (2n^2 - 1)k^2$.

Case 3: $dn(\xi, n)$ solution

Taking $\alpha = -k^2, \beta = (2 - n^2)k^2$, and $\gamma = -(1 - n^2)k^2$, the solution is:

$$\psi_3(x, y, t) = \left[\sqrt{\frac{-2k^2}{P_1}} dn(k\xi, n)\right] e^{i\theta}, \tag{33}$$

provided $P_2 = (2 - n^2)k^2$.

As $n \to 1$, the solutions ψ_1 and ψ_2 degenerate into dark and bright soliton solutions, respectively:

$$\psi_{n\to 1}: tanh(k\xi) \;\; or \;\; \text{sech}(k\xi). \quad (34)$$

This confirms that the Jacobi elliptic function method is consistent with the $(G'/G + A)$ - expansion method results, providing a unified framework for studying stochastic wave structures, the 2D-SCNLSE.

4. Results and discussion

In this section, we provide a detailed analysis of the numerical simulations and 3D surfaces of the analytical solutions extracted using MATLAB. The dynamics of wave propagation and amplitude behavior under the influence of stochastic perturbations were studied with different optically compatible noise densities and at a specific time $t = 0.5$. To ensure complete mathematical and physical compatibility with the diagonal two-dimensional wave transform hypothesis $\xi = \rho_1 x + \rho_2 y + \rho_3 t$, The software parameters were precisely adjusted to highlight the behavior of light and dark solitons, the flowing periodic waves of the first methodology, and the periodic behavior of the second methodology.

4.1. Results of the Generalized $(G'/G + A)$-expansion method

In this section, we turn to the analysis of the soliton solutions and periodic solutions generated by the first developed methodology. Solitons are characterized by concentration of amplitude in a narrow and specific spatial range, while the updated periodic solutions reflect extended chains of harmonious ripples affected by random environments added by wave transformation.

By reviewing Figure (1), we notice the pure geometric comparison between the three generated patterns. Section (a) explains the solution of the true bright soliton (Genuine Bright Soliton Profile) formed via the spatial derivative as an isolated, rising pulsating peak (Bell-shaped peak) that is efficiently centered and emanates from a stable and stable bottom level at the equilibrium level, where the combined group coefficients are set at (Δ= 1.5, $\rho_1 = 0.4$, $\rho_2 = 0$, $\rho_3 = -0.25$, $\sigma = 0.15$). On the other hand, Section (b) highlights the dark soliton solution (u_1 Profile), which represents a local gap or sharp drop in amplitude that descends steadily in straight valleys parallel to the spatial axes as a result of regular orientation and zeroing of the second component, with actual parameters of (Δ= 1.4, $\rho_1 = 0.64$, $\rho_2 = 0$, $\rho_3 = -0.4$, $\sigma = 0.15$). Concluding the visual comparison, Section (c) presents the Genuine Smooth Periodic Wave Solution, in which the anomalous sheared walls have completely disappeared thanks to the updated sinusoidal formulation. Here we notice the rotation and flow of the geometric structure of the waves to appear as oblique and frequent mountain ranges extending diagonally across the spatial network as a result of fully activating two-dimensional orientation with equal actual coefficient values for the two axes ($\rho_1 = \rho_2 = 0.175$, $\rho_3 = -0.14$) and with high noise intensity ($\sigma = 0.25$), which reflects the random appearance and the picturesque colour depth without a structural collapse of the solution. Physicist.

4.2. Results of the Jacobi Elliptic Function Method

This method is based on the class of Jacobi elliptic functions, and the solutions are projected into a planar spatial grid extending from -15 to 15 on the x-axis to ensure that the wave train is clearly accommodated, with the rotation coefficient of the second axis $\rho_2 = 0$ zeroed out to make the direction of wave propagation straight and parallel to the computational axes based on improving the visual perspective and the required vertical matching.

The figure (2) shows the three cases of elliptic solutions side by side, where branch (a) shows the sn-type periodic solution that is symmetrical around the zero equilibrium level and is prominent despite the surface wrinkles resulting from Gaussian noise $\sigma = 0.2$. While in section (b) the smooth sinusoidal solution of type cn-type is shown, which was produced under a negative system parameter $P_1 = -2$ to avoid imaginary values. Finally, the branch (c) represents the positive-definite dn-type wave structure that rises completely above zero in bell-like pulses, which as a whole reflects the stability and diversity of these elliptical solutions in turbulent environments.

The graphical representations of these solutions are displayed below in Figure 1 and Figure 2:

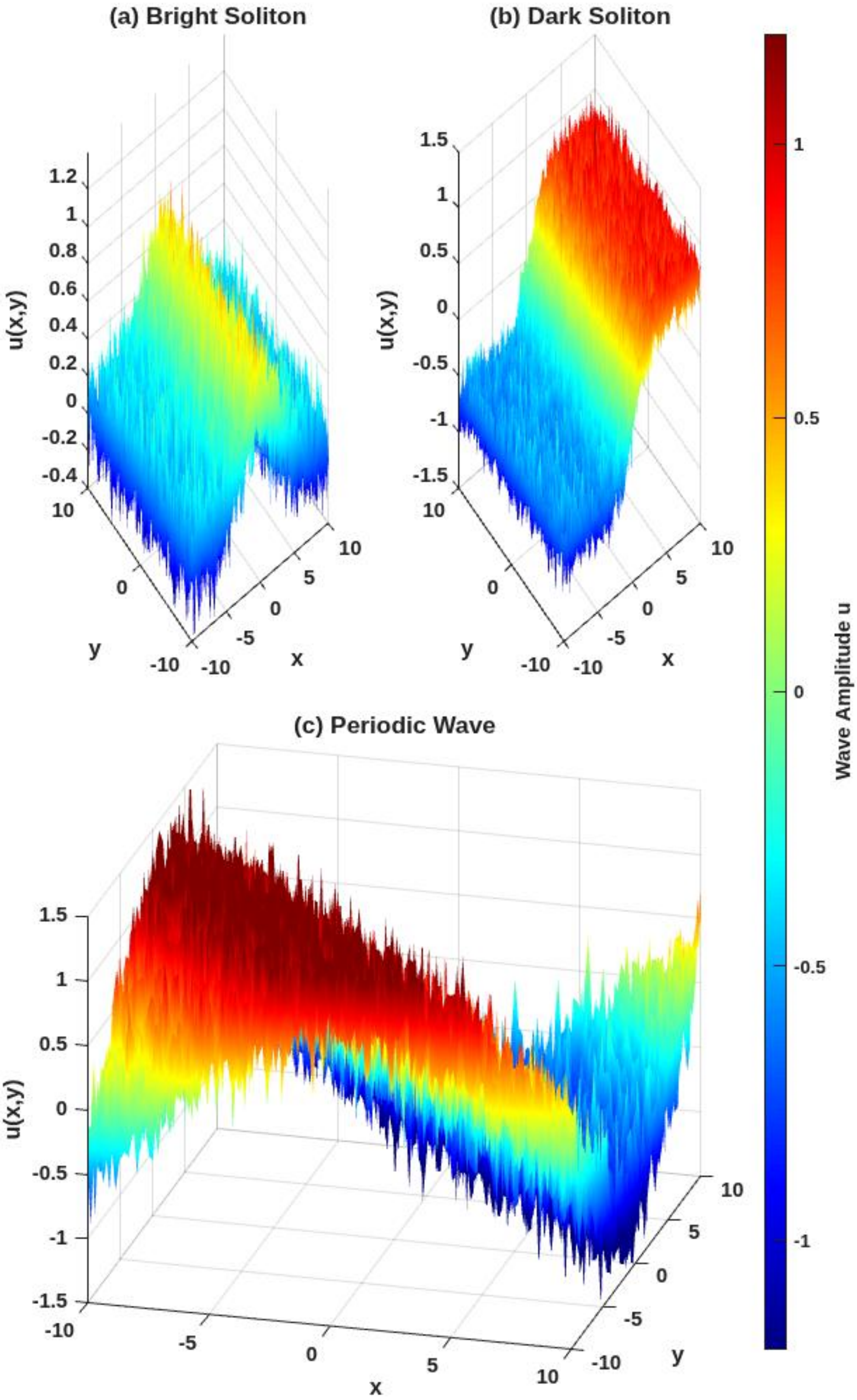

Figure 1: 3D numerical simulations of the solutions obtained via the enhanced (G′/G+A)-method under stochastic perturbations at a fixed time t = 0.5: **(a) Bright soliton profile** with parameters: $\Delta = 1.5$, $\rho_1 = 0.4$, $\rho_2 = 0$, $\rho_3 = -0.25$, and noise intensity $\sigma = 0.15$.**(b) Dark soliton profile** (u_1) with parameters: $\Delta = 1.4$, $\rho_1 = 0.64$, $\rho_2 = 0$, $\rho_3 = -0.4$, and noise intensity $\sigma = 0.15$.**(c) Smooth periodic wave** with parameters: $\rho_1 = \rho_2 = 0.175$, $\rho_3 = -0.14$, and noise intensity $\sigma = 0.25$.

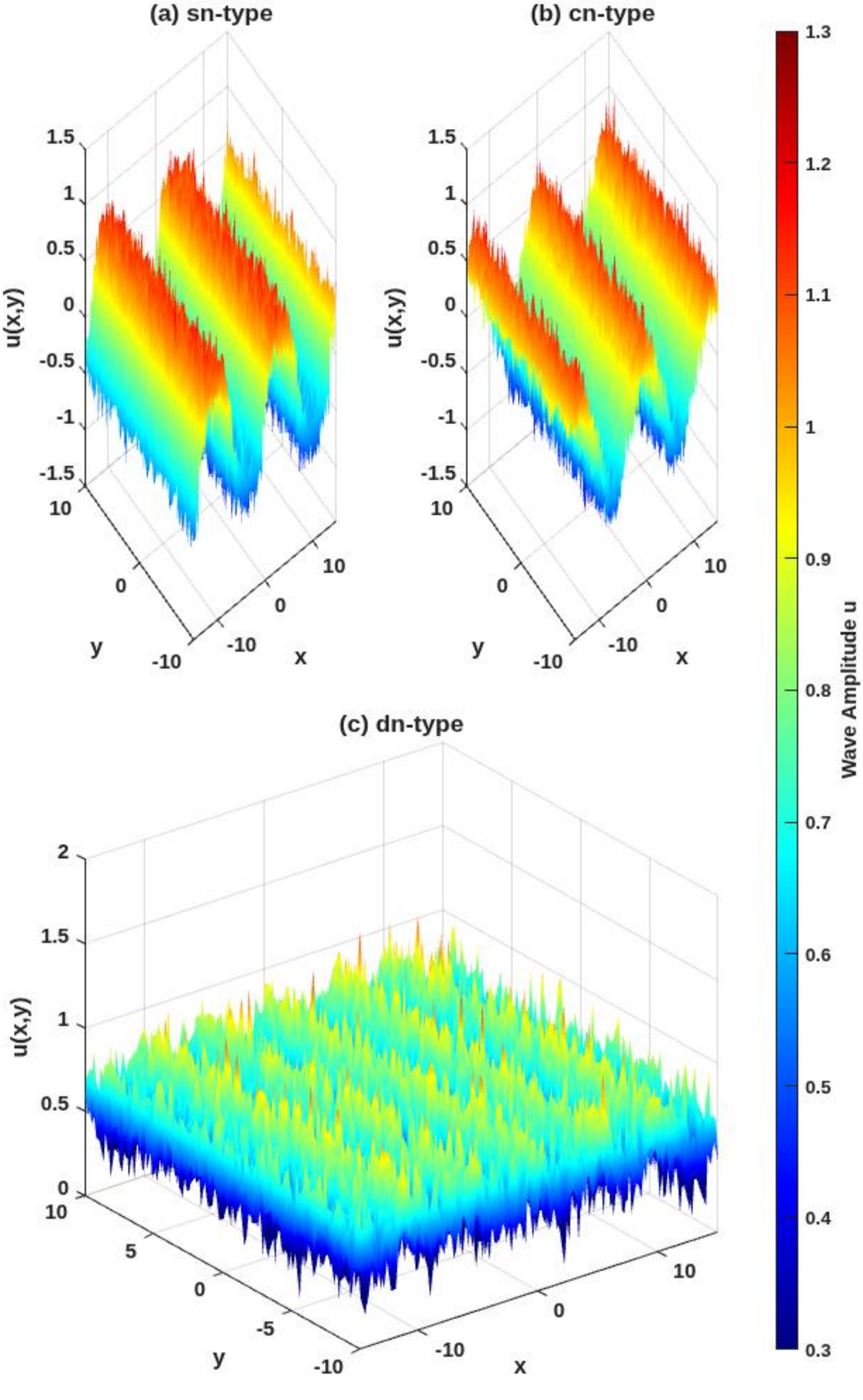

Figure 2: 3D numerical simulations of the solutions obtained via the Jacobi elliptic function expansion method under stochastic perturbations at a fixed time t = 0.5 and a linear spatial network $\rho_2 = 0$. **(a) *sn*-type solution** with parameters: $n = 0.85,\ P_1 = 2,\ \rho_1 = 0.7,$ and noise intensity $\sigma = 0.2$. **(b) *cn*-type solution** with parameters $n = 0.58,\ P_1 = -2,\ \rho_1 = 0.2$, and noise intensity $\sigma = 0.2$. **(c) *dn*-type solution** with parameters $n = 0.50,\ P_1 = -2,\ \rho_1 = 0$, and noise intensity $\sigma = 0.2$.

4.3. Comparison of Methodological Approaches

The results presented in this research show that the integration of the $(G'/G+A)$-expansion and Jacobi elliptic equation methods creates an extensive system for determining precise solutions to the (2+1)-dimensional stochastic CNLSE. In contrast to earlier deterministic studies [5, 6, 7], the current work includes multiplicative noise through a stochastic phase component, offering a more faithful representation of wave dynamics in irregular media. Localized Structures: The $(G'/G+A)$-expansion technique is efficient at producing bright and dark solitons as well as rational solutions, demonstrating the balance of dispersion and nonlinearity. Periodic Transitions: The Jacobi elliptic approach yields sn, cn, and dn type solutions that transform into hyperbolic solitons when n reaches 1, connecting periodic and solitary structures.

4.4. Physical Interpretation of Stochastic Effects

In stochastic terms, noise intensity σ mainly affects phase behavior, resulting in stochastic displacements and minor amplitude variations. Crucially, the soliton's core structure stays intact under moderate noise. This resilience aligns with findings in plasma physics and fiber optics, where external noise impacts phase consistency but leaves nonlinear wave patterns unharmed. Ultimately, while localized formations are identified by the $(G'/G+A)$ technique, periodic movements are characterized by the Jacobi method, broadening the stochastic CNLSE solution range to act as a reference for subsequent studies [9, 10, 11].

5. Conclusion

This study concluded by successfully analyzing the coupled nonlinear Schrödinger equation (CNLSE) with dimensions (2+1) in a stochastic context. The validity and reliability of the results were confirmed through several indicators: It was shown that the inferred elliptic Jacobi solutions automatically lead to the classical soliton solutions when approaching the limit $n \to 1$, which ensures their compatibility with the known deterministic models.

The general expansion method (G′/G+A) has also proven its efficiency in generating distinct solution systems that are fully consistent with the auxiliary equations and previous literature in the case (A = 0). Moreover, the results showed that despite the multiplicative noise effect, the stability of the wave patterns reflects a precise and stable balance between nonlinearity and dispersion forces.

Based on the above, this study confirms the effectiveness of the proposed analytical framework in studying nonlinear media subject to random effects.

Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.

References

[1] P. F. Byrd and M. D. Friedman, Handbook of Elliptic Integrals for Engineers and Scientists, Springer, 1971.
[2] P. G. Drazin and R. S. Johnson, Solitons: An Introduction, Cambridge University Press, 1989.
[3] W. Malfliet, "Solitary wave solutions of nonlinear wave equations," Am. J. Phys., vol. 60, pp. 650–654, 1992.
[4] C. Sulem and P. L. Sulem, The Nonlinear Schrödinger Equation: Self-Focusing and Wave Collapse, Springer, 1999.
[5] M. Wang, X. Li, and J. Zhang, "The G′/G-expansion method and traveling wave solutions of nonlinear evolution equations," Physics Letters A, vol. 372, pp. 417–423, 2008.
[6] M. N. Alam and M. A. Akbar, "Traveling wave solutions via the new generalized (G′/G)-expansion method," SpringerPlus, vol. 3, p. 122, 2014
[7] A. Bekir and Ö. Güner, "Exact solutions of nonlinear fractional differential equations by (G'/G)-expansion method," Chinese Physics B, vol. 22, no. 11, p. 110202, 2013.
[8] M. El-Horbaty, K. A. Gepreel, and Y. Yildirim, "Investigation of optical soliton solutions for the cubic-quartic derivative nonlinear Schrödinger equation using advanced integration techniques," Physica Scripta, vol. 99, no. 11, p. 115209, 2024.
[9] S. Albosaily, W. W. Mohammed, M. A. Aiyashi, and M. A. E. Abdelrahman, "Exact Solutions of the (2+1)-Dimensional Stochastic Chiral Nonlinear Schrödinger Equation," Symmetry, vol. 12, no. 11, p. 1874, 2020.
[10] M. A. E. Abdelrahman and W. W. Mohammed, "Analytical solutions of stochastic nonlinear evolution equations using the G'/G-expansion method," Journal of Applied Mathematics and Physics, vol. 9, pp. 1503–1516, 2021.
[11] Y. Zhang, H. Wang, and L. Li, "Stochastic effects on nonlinear Schrödinger equation with multiplicative noise," Nonlinear Dynamics, vol. 95, pp. 2565–2576, 2019.
[12] Y. S. Kivshar and B. Luther-Davies, "Dark optical solitons: physics and applications," Physics Reports, vol. 298, pp. 81–197, 1998.

