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Study of the internal didactic transposition of the concept of reasoning by mathematical induction in Tunisian secondary education

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دراسة النقل التعليمي الداخلي لمفهوم البرهان بالاستقراء الرياضي في التعليم الثانوي التونسي

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Abstract:
This paper focuses on the teaching of reasoning by mathematical induction in Tunisian secondary schools. The main objective is to analyse mathematics teachers' knowledge of this fundamental concept and how they approach it in the classroom. The study reveals several significant findings, among which is the often-superficial presentation of this reasoning in the curriculum. This limited approach has an impact on the effective transmission of this complex concept to learners.

Keywords: Mathematical induction, syntactic, semantic, school textbook, internal didactic transposition.

الملخص
تركّز هذه الورقة البحثية على تدريس مفهوم الاستقراء الرياضي في المدارس الثانوية التونسية. ويتمثل الهدف الرئيسي في تحليل معرفة مدرّسي الرياضيات بهذا المفهوم الأساسي وكيفية تناولهم له في الفصل الدراسي. وتكشف الدراسة عن العديد من النتائج المهمة، من بينها العرض السطحي غالبًا لهذا التفكير الرياضي في المناهج الدراسية. هذا النهج المحدود له تأثير على النقل الفعال لهذا المفهوم المعقد إلى المتعلمين.

الكلمات المفتاحية: الاستقراء الرياضي، النحوي، الدلالي، الكتاب المدرسي، النقل التعليمي الداخلي.

Introduction

Reasoning by mathematical induction plays a dual role in mathematics: it serves both as a tool for constructing mathematical objects and as the basis for many demonstrations in discrete mathematics (Grenier, 2012). However, several studies (Gardes, Gardes and Grenier, 2016; Soltani, 2019; Soltani and Chellougui, 2023) highlight that learners encounter major cognitive difficulties with this type of reasoning, which can be grouped into three main categories: technical, mathematical and conceptual. These difficulties are attributed to various factors, in particular the way in which reasoning by mathematical induction, a complete form of induction, is taught. Often reduced to a simple mechanical and algebraic manipulation, it is rarely presented as a concept in its own right. More specifically, Dogan (2016) observes that students have difficulty articulating basis step and inductive step mathematical induction, while Harel (2002) sees confusion between intuitive induction and mathematical proof. This approach limits the understanding of students, who perceive reasoning either as a poorly assimilated method of proof (Grenier, 2012; Gardes, Gardes and Grenier, 2016), or as a simple algorithm with no real mathematical meaning (Soltani, 2019; Soltani, 2025).

Our aim is to explore the origins of these difficulties in order to gain a better understanding of the challenges involved in teaching reasoning by mathematical induction. To do this, we are interested in the internal didactic transposition (Chevallard, 1985), namely the transformation of knowledge to be taught into taught knowledge (Figure 1). We try to shed light on how this transformation takes place in the context of teaching reasoning by mathematical induction, a fundamental concept in mathematics

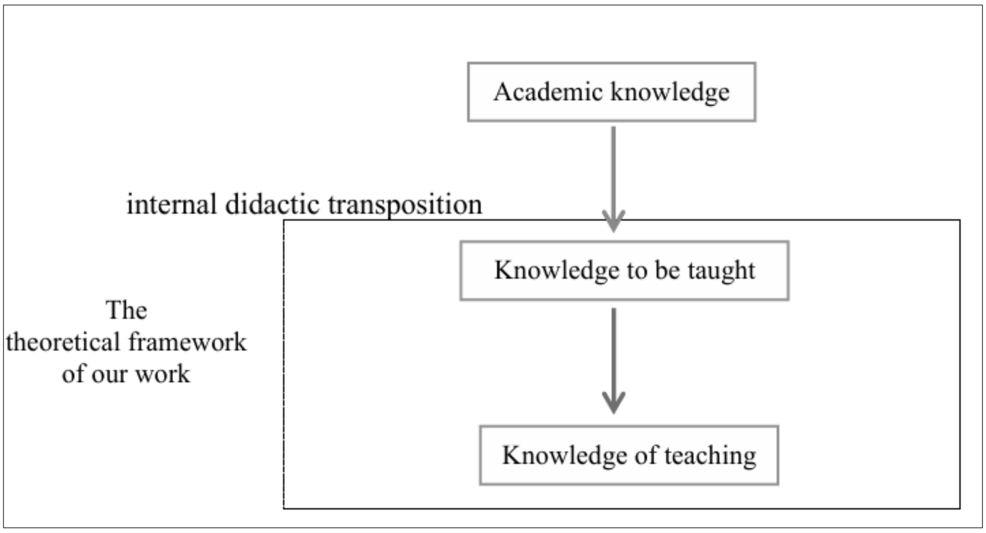

Figure 1. Diagram of Chevallard's (1985) internal didactic transposition.

In our study, we consider that the *knowledge to be taught* is constituted by the official curriculum and school textbooks[1], whereas the *knowledge taught (of teaching)* corresponds to what is actually transmitted in class, i.e. the knowledge that a teacher actually presents to pupils.

To this end, we have undertaken an in-depth analysis of Tunisian school curricula and textbooks, focusing on the way in which reasoning by mathematical induction is dealt with in official secondary school curricula and textbooks. This step aims to assess the consistency between theoretical knowledge, as presented in the pedagogical documents, and its application in class. Next, we examined teachers' practices by analysing the proofs and reasoning they write, using specific extracts from the extracurricular mathematics textbook: CMS[2] (4th year, Math section). This assessment was carried out using the five analysis criteria developed by Gardes, Gardes and Grenier (2016). These are: *Structure of the reasoning; Explanation and notation of the property depending on the natural number n; Basis step; Inductive step: implication and quantification and Structure of the conclusion*. These criteria make it possible to examine the clarity of explanations, the logical rigor of proofs, as well as compliance with syntactic and semantic rules.

The value of this approach lies in its ability to reveal not only any shortcomings in the presentation of knowledge in textbooks, but also to highlight the obstacles encountered by teachers in transmitting abstract concepts such as reasoning by mathematical induction.

1. Reasoning by mathematical induction in the officiel Tunisian secondary mathematics curriculum and textbooks

1.1. Reasoning by mathematical induction in the official mathematics program

Reasoning by mathematical induction occupies an essential place in the official secondary mathematics curriculum in Tunisia, in force since the 2008 reform (*3rd year of secondary education, sections Mathematics, Experimental Sciences, Technical Sciences and Computer Science*). This program emphasizes the development of three key skills: *practicing a mathematical approach, deploying mathematical reasoning*, *and communicating using mathematical language*. These skills are interconnected and form the basis for learning in mathematics.

[1] In Tunisia, the textbook is an official document.

[2] The parallel textbook provides detailed corrections developed by two lead teachers for the 2007-2008 period, covering all exercises from both official Year 4 Mathematics textbooks (Volume 1 and Volume 2). It serves as a reference for both learners and teachers.

The program (2008) proposes a typology of different mathematical reasoning, *including inductive*, *deductive*, by *absurdity* and by *mathematical induction*, with the main objective of leading students to produce rigorous evidence, to structure and justify each step of a relevant approach. It also states that: « *Through written or oral activities, pupils will develop their ability to research, experiment, conjecture and verify a result. They will also consolidate their inductive, deductive, absurdist and recursive reasoning chains*. »[3]. (DPN[4], 2008, p. 3).

However, despite the importance of logic in mathematics, it is notable that this concept is not explicitly addressed in the curriculum. Although logical connectors such as implication and equivalence, as well as quantifiers, yet essential to mathematical activity, they are absent from the curriculum. This omission can hinder students' understanding of statements and the development of rigorous reasoning. This has been confirmed by Chellougui (2003), whose research shows that the lack of explicit instruction in quantification within Tunisian secondary education leads to significant difficulties in students' mathematical reasoning.

For 3rd year students (*Mathematics, Experimental Sciences, Technical Sciences and Computer Sciences sections*), reasoning by mathematical induction is explicitly presented. Indeed, the disciplinary content is systematically structured for each section with particular emphasis on the skills that students need to develop.

Details of how this reasoning is taught are shown in the table below (Table 1), based on 2008 mathematics.

Table 1. Reasoning by mathematical induction in the mathematics syllabus for the 3rd year of secondary school (2008, pp. 33-65).

Section	Part of the curriculum	subject content	skills to be developed
Mathematics	Analysis	Numerical sequences	The pupils use a technique, an algorithm or a procedure to: *Exploit the mathematical induction to show that a real number is a majorant or a minorant of a sequence or to study the variations of a sequence.*
	Arithmetic and Combinatorics	Principle of mathematical induction.	The pupils use a technique, an algorithm or a calculation procedure to *:* *Prove a property about natural numbers using the mathematical induction.*
Exprimental Sciences	Analysis	-Principle of mathematical induction. - Numerical sequences	The pupils use a technique, an algorithm or a procedure to: *Exploit the mathematical induction to show that a real number is a majorant or a minorant of a sequence or to study the variations of a sequence.*
Technical Sciences	Analysis	Principle of mathematical induction.	*Know the limit of a geometric sequence.* *Give the fractional form of a rational knowing its periodic unlimited decimal expansion.*
Computer Sciences sections	Logic, arithmetic and numeration systems	Principle of mathematical induction	-The pupils use a technique, an algorithm or a procedure to : *Prove a property about integers using reasoning by mathematical induction.* -The pupils solve problems in mathematical situations or in relation to the environment. In particular: *They develop their mathematical reasoning skills.*

[3] Our translated from French.

[4] National Directorate for Education

As illustrated in the table below (Table 1), reasoning by mathematical induction occupies a central place in the disciplinary content of the various sections of 3rd year secondary school. Referred to as the principle of mathematical induction, it is taught in several key mathematical areas: Analysis, Arithmetic and Enumeration, Arithmetic Logic and Numbering Systems. This diversity of applications shows the versatility of reasoning by mathematical induction, not only as a method of proof, but also as a tool for structuring mathematical thought. By covering several areas in this way, this reasoning enables learners to tackle a variety of concepts using a common method, thereby promoting the transferability of the skills acquired.

1.2. Presentation of reasoning by mathematical induction in 3rd year secondary school mathematics textbooks

Reasoning by mathematical induction in the 3rd year textbook (Mathematics section)

In the textbook of the Mathematics section, reasoning by mathematical induction is integrated into the chapter *Divisibility in IN*, in the paragraph entitled "*Principle of mathematical induction*" as follows:

Principle of mathematical induction:
Let n_0 be a natural number and P_n a property depending on a natural number n greater than or equal to n_0.
If the following two conditions hold:

- P_{n_0} is true,
- if P_n is true then P_{n+1} is true,

then P_n is true for all n greater than or equal to n_0.

Figure 2. Extract from the textbook *3rd year secondary, Mathematics section*, Volume2. Page 128, translated into English.

This principle (Figure 2) presents itself as a method to demonstrate that a property, dependent on a natural number, is true from a certain rank. The presentation follows the traditional stages of reasoning by mathematical induction: basis step and inductive step. However, the notation used for the property to be demonstrated, P_n, although commonly used for sequences, does not correctly introduce the variable n, and the inductive step condition, formulated as «*If P_n is true then P_{n+1} is true*», lacks syntactic and semantic rigour. This formulation reduces the inductive step to a simple verification of the truth of P_{n+1}, obscuring the fact that it is actually a demonstration of a universal implication (Gardes, Gardes and Grenier, 2016). This presentation gives the reasoning an algorithmic character that is lacking in terms of mathematical rigour. (Soltani, 2019; Soltani, 2025).

Reasoning by mathematical induction in the 3rd year textbook (Experimental Sciences section)

In the Experimental Science section textbook, reasoning by mathematical induction appears in the chapter on *Real sequences*, with a presentation similar to that of the 3rd year Mathematics section textbook. It is also entitled *Principle of mathematical induction*, and its approach remains the same: a two-stage demonstration (basis step and inductive step), with the same shortcomings observed in terms of formalisation and rigour.

Reasoning by mathematical induction in the 3rd year textbook (Technical sciences section)

The manual of the Technical Sciences section also addresses reasoning by mathematical induction in the chapter entitled *Real Sequences* and offers a detailed explanation in the section *Reasoning by mathematical induction*, under the heading Courses. This reasoning is formulated in the form of a principle as follows:

THE PRINCIPLE:
Let n_0 be a given natural number.
To prove by induction that a property P(n) related to a natural number n is true for all $n \geq n_0$, we must:

- Prove that the property holds for n_0 (initialization),
- Prove that if the property is true for a rank $n \geq n_0$, then it remains true for the next rank *(n+1)* (the property is hereditary).

Figure 3. Extract from the textbook *3rd year secondary, Technical sciences section*, page 113, translated into English.

This principle (Figure 3) presents reasoning by mathematical induction in two stages, namely basis step and inductive step. However, several syntactic problems persist, particularly in the introduction of the variable n and the formulation of inductive step. The latter does not make it clear that it should correspond to a universal implication, thus masking a crucial stage in the reasoning (Gardes, Gardes and Grenier, 2016). Moreover, the conclusion stage is implicitly mentioned from the beginning of the reasoning, which makes the logical relationship between the three fundamental stages of reasoning by mathematical induction ambiguous. The conclusion, which should be the direct consequence of basis step and inductive step, appears in a less than explicit form, thus hiding the true implication between these different phases.

The authors of this manual have added a remark to clarify this type of reasoning, illustrated in the following figure (Figure 4):

Remark:
The principle of mathematical induction can be illustrated with the image of a ladder :
If you can :
. get on the first rung of the ladder,
and
. move from any given rung to the next one,
Then you can climb every rung of that ladder

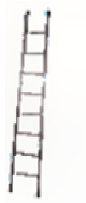

Figure 4. Extract from the textbook, *3rd year secondary, Technical Sciences section,* page 113, translated into English.

However, as Gardes, Gardes and Grenier (2016) note, this illustration, while useful, fails to capture some fundamental aspects of the mathematical induction principle, and the approach often remains too mechanical and insufficiently conceptual. As a result, reasoning by mathematical induction is still perceived more as a proof algorithm than as a genuine tool for rigorous mathematical reflection. (Soltani, 2019; Soltani, 2025).

Reasoning by mathematical induction in the 3rd year textbook (Computer Science section)
In the 3rd year textbook (Computer Science section), reasoning by mathematical induction occupies an important place in the Arithmetic chapter. It is presented explicitly in a first paragraph entitled *Reasoning by mathematical induction*, as follows:

Reasoning by mathematical induction:
Theorem (admitted)
Let P_n be a property that depends on the natural integer n. If we have:
1) The property P_k is true ($k \in \mathbb{N}$).
2) For every natural integer n such that $n \geq k$, (P_n is true $\Rightarrow$ P_{n+1} is true).
Then for every natural integer n such that $n \geq k$, P_n is true.

Figure 5. Extract from the textbook, *3rd year secondary, Computer Science section*, page 285, translated into English.

This reasoning (Figure 5) is introduced to demonstrate that a property dependent on a natural number is true from a certain rank, through two stages: basis step and inductive step of reasoning by mathematical induction.

However, there are shortcomings in mathematical formulation, particularly in the notation of the P_n property and in the use of variables. The variables n and k are introduced simultaneously, creating syntactic and semantic ambiguity. These variables are not quantified and remain free variables. The expression «a property depends on the integer n» implies that n is a variable fixed beforehand. Furthermore, the final stage of the reasoning, which concerns the conclusion of the process by mathematical induction, is presented without reference to the variable k.

Discussion
It should be noted that although the curriculum includes mathematical induction in important parts of the content, its teaching often remains focused on technical aspects, with limited attention to deep understanding of the principle as a mathematical concept.

2. Analysis of teacher productions in the corrected mathematics manuel (CMS- 4th year secondary, Volume1) related to mathematical induction Based reasoning

In this section, we analyse the corrections made by two teachers to exercise questions requiring reasoning by mathematical induction in the CMS after-school textbook (4th year, Math section, Volume 1). We chose three specific questions[5] to evaluate the structure of the proposed proofs. This analysis is based on the work of Gardes, Gardes and Grenier (2016), Soltani and Chellougui (2023), and Soltani (2025), who detail the key stages of reasoning by mathematical induction: the notation of the property as a function of n, basis step, inductive step and conclusion.

Question 1

This is *question 2* of exercise 15[6], p. 48, in the chapter *Real sequences* in the 4th year textbook (Mathematics section, Volume 1).

Attached is the proposed answer to this question in the CMS extracurricular manual:

2) par reccurence

- pour n=0 : $U_0 = 2\cos\theta = 2\cos(\frac{\theta}{2^0})$ (vrai)
- supposons que : $U_n = 2\cos(\frac{\theta}{2^n})$
- montrons que : $U_{n+1} = 2\cos(\frac{\theta}{2^{n+1}})$

$U_{n+1} = \sqrt{2+U_n} = \sqrt{2(1+\cos(\frac{\theta}{2^n}))} = \sqrt{4.\cos^2(\frac{\theta/2^n}{2})}$

$= 2\left|\cos(\frac{\theta}{2^{n+1}})\right| = 2\cos(\frac{\theta}{2^{n+1}})$ car $\frac{\theta}{2^{n+1}} \in \left]0, \frac{\pi}{2}\right[$

conclusion: $U_n = 2\cos(\frac{\theta}{2^n})$; $\forall n \in \mathbb{N}$

English translation

2) by mathematical induction

. for n =0: $U_0 = 2\cos\theta = 2\cos(\frac{\theta}{2^0})$ (true)

. Let's assume that: $U_n = 2\cos(\frac{\theta}{2^n})$

. Let us demonstrate that: $U_{n+1} = 2\cos(\frac{\theta}{2^{n+1}})$

$U_{n+1} = \sqrt{1+U_n} = \sqrt{2(1+\cos(\frac{\theta}{2^n})} = \sqrt{4.\cos^2(\frac{\frac{\theta}{2^n}}{2})}$

$= 2\left|\cos(\frac{\theta}{2^{n+1}})\right| = 2\cos(\frac{\theta}{2^{n+1}})$ because $\frac{\theta}{2^{n+1}} \in \left]0, \frac{\pi}{2}\right[$

Conclusion: $U_n = 2\cos(\frac{\theta}{2^n})$; $\forall$ n∈ IN.

Figure 6. Extract from the answers to question 2 of exercise 15, CMS extracurricular manual, (Volume 1), p.20.

In the extract from the answer key (Figure 6), the property to be demonstrated is not identified. The basis step is not validated, because the property to be demonstrated is from rank 1 and not rank 0. The inductive step is also insufficiently justified: the implication is implicit and there is no quantification of the variable n, which obscures the demonstration of universal implication. The conclusion, on the other hand, is identified in a way that does not conform to logical syntax, where quantification is placed at the end of the sentence. This is standard practice in textbooks and in teachers' work (Chellougui, 2020). We also note that in this conclusion the domain considered is the IN set, which is consistent with basis step, but not consistent with the statements of question 2.

Question 2

This is *question 4.b.* of exercise 20[7] on p. 49, in the same chapter *Real sequences*.

Attached is the proposed answer to this question in the CMS extracurricular manual:

4) $\begin{cases} x_0 = 2 \\ x_{n+1} = f(x_n) \end{cases}$

b) par reccurence

- $U_0 = 2 \in \mathbb{Q}_+$ (vrai)
- supposons que $x_n \in \mathbb{Q}_+$,

montrons que $x_{n+1} \in \mathbb{Q}_+$

$x_{n+1} = f(x_n) = 1 + \frac{1}{x_n} \in \mathbb{Q}_+$

English translation

4) $\begin{cases} x_0 = 2 \\ x_{n+1} = f(x_n) \end{cases}$

b) by mathematical induction

* $x_0 = 2 \in \mathbb{Q}_+$ (true)

* Let's assume that: $x_n \in \mathbb{Q}_+$

Let us demonstrate that $x_{n+1} \in \mathbb{Q}_+$

$x_{n+1} = f(x_n) = 1 + \frac{1}{x_n} \in \mathbb{Q}_+$

Figure 7. Extract from the answers to question 4.b. of exercise 20, CMS Extracurricular Manual, (Volume 1), p. 22

[5] These three questions are taken from the following exercises in the 4th year textbook (Mathematics section): Exercise 15 page 48 'Volume 1', Exercise 20 page 49 'Volume 1' and Exercise 17 page 176 'Volume 2'.

[6] See appendix

[7] See appendix

In the excerpt from the correction (Figure 7), the property to be demonstrated is not identified, and the basis step is not explicitly verified. The inheritance step also lacks justification, the implication being implicit. As in the previous exercise, the inheritance step is divided into two sub-steps (indicated by asterisks), without rigorous demonstration and without relevant explanation. Moreover, the conclusion of reasoning by mathematical induction is completely absent.

Question 3

We are interested in *question 1. a.* of exercise 17 p. 176, from the chapter *Identity of Bezout*, from the manual of the 4th year (section Mathematics, Volume 2). The suggested answers can be found on page 205 of the CMS extracurricular manual:

1\. a/ * <u>Pour n=1</u> $u^n = u^1 = u = 2+\sqrt{3} = a_1+b_1\sqrt{3}$ avec $a_1 = 2$ et $b_1 = 1$ deux entiers naturels

Donc la propriété est vrai à l'ordre initial

* <u>Soit n∈IN*</u> Supposons que $u^n = a_n+b_n\sqrt{3}$ avec a_n et b_n deux entiers et montrons que $u^{n+1}=a_{n+1}+b_{n+1}\sqrt{3}$ avec a_{n+1} et b_{n+1} deux entiers

On a $u^n= a_n+b_n\sqrt{3} \Rightarrow u^{n+1} = u.u^n = (2+\sqrt{3}).(a_n+b_n\sqrt{3}) = (2a_n+3b_n)+(2b_n+a_n)\sqrt{3}$

$= a_{n+1} + b_{n+1}\sqrt{3}$

Comme a_n et b_n sont deux entiers naturelles alors $a_{n+1}=2a_n+3b_n$ et $b_{n+1} = 2b_n+a_n$ sont deux entiers naturelles

Cl : <u>pour tout n∈IN* $u^n= a_n+b_n\sqrt{3}$ avec a_n et b_n deux entiers naturelles</u>

English translation

1.a/ ⁎ for n =1 $u^n = u = 2+\sqrt{3} = a_1 + b_1\sqrt{3}$ with a_1=2 and b_1 =1 two natural integers

So, the property is true in the initial order

⁎ Let n ∈ IN* Let's assume that $u^n = a_n + b_n\sqrt{3}$ with an and bn two integers and let us demonstrate that $u_{n+1} = a_{n+1} + b_{n+1}\sqrt{3}$ with a_{n+1} and b_{n+1} two integers

We have $u_n = a_n + b_n\sqrt{3} \Rightarrow u^{n+1} = u.\, u^n$

$= (2+\sqrt{3})(a_n + b_n\sqrt{3}) = (2a_n + 3b_n) + (2a_n + 3b_n)\sqrt{3}$

$= a_{n+1} + b_{n+1}\sqrt{3}$

As a_n and b_n two naturel integers then $a_{n+1}= 2a_n+3b_n$ and $b_{n+1}= 2b_n+a_n$ are two naturel integers

Cl: <u>for everything n ∈ IN* $u^n = a_n + b_n\sqrt{3}$</u> with <u>a_n and b_n two naturel integers.</u>

Figure 8. Extract from the answers to question 1.a. of exercise 17, CMS extra-curricular manual, (Volume 1), p. 205

In this extract from the answers (Figure 8), the first major flaw is the failure to identify the property to be demonstrated. The basis step is poorly justified, especially with the formulation « *So the property is true to the initial order* », without the property in question being specified. This sentence thus loses all its meaning, constituting a semantic error (Soltani and Chellougui, 2023; Soltani, 2025). Furthermore, the consequence of the inductive step is omitted: « *for any integer n ≥ 1, (P(n) implies P(n+1)) is true*» (Gardes, Gardes and Grenier, 2016). This compromises the rigour of the proof. However, the conclusion is well identified.

Discussion

The presentation of reasoning by *mathematical induction* in official secondary school textbooks is often superficial. This is particularly apparent in what teachers produce when correcting exercises, as in the *CMS extracurricular mathematics textbook, 4th year, Mathematics section (Volume 1).* An analysis of the structure of this reasoning reveals: a total absence of the formulation of the property to be demonstrated; ambiguities in the verification of the basis step stage; demonstrations that are not correctly justified in the inductive step stage, as well as implicit implication and quantification; the conclusion stage is most often identified.

Results and Conclusion

The analysis of curricula and teachers' work reveals a lack of familiarity with reasoning by mathematical induction. The inheritance stage proves to be particularly difficult for some teachers to master, especially regarding the formulation of the property to demonstrate as well as the correct use of the implication and universal quantifier. These shortcomings are often linked to a lack of knowledge of logic. (Chellougui, 2009 ; Fabert et Grenier, 2011 ; Grenier, 2012 ; Mesnil, 2014).

The results of this study highlight the inherent complexity in teaching reasoning by mathematical induction, particularly in the context of secondary education. It is now imperative to pay greater attention to the training of mathematics teachers, by insisting on the fundamental concepts of language, elements of logic and mathematical reasoning. A better grasp of these concepts, both theoretically and practically, would not only improve the quality of mathematical induction teaching, but also clarify the crucial steps underlying it (Soltani, 2025).

This training should aim to reinforce a conceptual approach to teaching reasoning by mathematical induction. Rather than reducing this method to a mechanical procedure, it is crucial to develop in students a deep and intuitive understanding of recurrent reasoning. This would allow them to apply it more rigorously in various mathematical situations, thus promoting greater autonomy in solving complex problem.

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Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.

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Appendix

Exercise 15, p. 48, 4th year textbook, Mathematics section, (Volume 1)

15 Soit θ un réel appartenant à $]0, \frac{\pi}{2}]$.

On considère la suite (u_n) définie par

$u_0 = 2\cos\theta$, $u_{n+1} = \sqrt{2+u_n}$, $n \geq 1$.

1. Exprimer u_1 et u_2 en fonction de θ.
2. Montrer que pour tout $n \geq 1$, $u_n = 2\cos\left(\frac{\theta}{2^n}\right)$.
3. Calculer $\lim\limits_{n\to+\infty} u_n$.

English translation

Let θ be a real number in $\left]0, \frac{\pi}{2}\right[$

Consider the sequence (u_n) defined by

$U_0 = 2\cos\theta$, $u_{n+1} = \sqrt{1+u_n}$, $n \geqslant 1$.

1. Express u_1 and u_2 in terms of θ.
2. Show that for all $n \geqslant 1$, $u_n = 2\cos(\frac{\theta}{2^n})$
3. Calculate $\lim\limits_{n\to+\propto} u_n$

Exercise 20 p. 49, 4th year textbook, Mathematics section, (Volume 1)

20 Soit la fonction f définie sur $]0,+\infty[$ par $f(x) = 1 + \frac{1}{x}$.

1. Etudier les variations de f et déterminer $f(]0,+\infty[)$.
2. Représenter la fonction f dans un repère $(O, \vec{i}, \vec{j})$, ainsi que la droite d'équation $y = x$.
3. On note $\varphi = \frac{1+\sqrt{5}}{2}$, montrer que $f(\varphi) = \varphi$.
4. On se propose de construire une suite (x_n) de rationnels qui converge vers φ.

On pose $x_0 = 2$ et $x_{n+1} = f(x_n)$, $n \geq 0$.

a. Calculer et représenter les réels x_1, x_2, x_3 et x_4.

b. Montrer que pour tout entier n, x_n est un rationnel positif.

c. Montrer que $|x_{n+1} - \varphi| \leq \frac{4}{9}|x_n - \varphi|$.

d. Conclure.

English translation

Let f be the function defined on $]0, +\infty[$ by

$f(x) = 1 + \frac{1}{x}$.

1. Study the variations of f and determine $f(]0, +\infty[)$
2. Plot the function f in the Cartesian coordinate $(O, \vec{\imath}, \vec{\jmath})$ along with the line $y = x$.
3. Let $\varphi = \frac{1+\sqrt{5}}{2}$, show that $f(\varphi) = \varphi$.
4. We aim to construct a sequence (x_n) of rational numbers converging to φ.

Set $x_0 = 2$ and $x_{n+1} = f(x_n)$, $n \geq 0$.

a. Calculate and write down the values of the real numbers x_1, x_2, x_3 and x_4.

b. Show that for all integers n, x_n is a positive rational number.

c. Show that $|x_{n+1} - \varphi| \leq \frac{4}{9}|x_n - \varphi|$ for all n.

d. Conclude.

Exercise 17 p. 176, , 4th year textbook, Mathematics section, (Volume 2)

	English translation
17 On pose $u = 2+\sqrt{3}$. 1. a. Montrer par récurrence que, pour tout entier naturel $n \geq 1$, $u^n = a_n + b_n\sqrt{3}$, où a_n et b_n sont des entiers naturels. b. Exprimer a_{n+1} et b_{n+1} à l'aide de a_n et b_n. 2. a. Montrer que $a_n{}^2 - 3b_n{}^2 = 1$ et $a_n b_{n+1} - a_{n+1} b_n = 1$. b. En déduire que $a_n \wedge b_n = a_{n+1} \wedge a_n = b_{n+1} \wedge b_n = 1$.	Let $u = 2+\sqrt{3}$. 1.a. Prove by mathematical induction that for all natural integers, $n \geq 1$, $u_n = a_n + b_n\sqrt{3}$ where a_n and b_n two natural integers. b. Express a_{n+1} and b_{n+1} in terms of a_n and b_n 2.a. Show that $a_n^2 - 3b_n^2 = 1$ and $a_n b_{n+1} - a_{n+1} b_n = 1$ b. Deduce that: $a_n \wedge b_n = a_{n+1} \wedge a_n = b_{n+1} \wedge b_n = 1$

